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WEDGE-BEAM FRAMING

By Arsham Amirikian, M. ASCE

STRUCTURAL DIVISION

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PAPERS

WEDGE-BEAM FRAMING

By Arsham Amirikian,[1] M. ASCE

Synopsis

Appreciable savings in materials and in the cost of a structural framing can be assured by the use of elements having tapering depth or flanges. "Wedge beams" can be substituted for members of constant cross section, as normally utilized in conventional framing, by introducing a series of hinges or joints of limited moment capacity. In a rectangular framing, the hinges are placed at midspan of the horizontal members and at the base of the columns, and the sections are tapered from a maximum at the rigidly connected joints to a minimum at the hinges.

The analysis of the "wedge-beam" framing remains in the domain of statically indeterminate structures. However, in comparison with rigid or continuous framing, it involves the solution of a relatively small number of unknowns, consisting of hinge reactions in the alternate bays of a given frame.

In this paper suggested arrangements of wedge-beam framing are outlined, the range of application is discussed, and a method of analysis is presented. Formulas and tables, as well as a simplified procedure of solution of simultaneous equations, are also given to facilitate the necessary computations in practice.

Introduction

The flexural pattern of a member in conventional arrangement of framing, utilizing either rigid-end or simple-end connections, follows an irregular course. The variations occur not only in the shape of the bending moment diagram but in the location of the maximum bending moment as well, in accordance with the condition and the position of the imposed loading. In the case of rigid framing, the flexural pattern is further affected by the changes in the relative stiffness of the various members. Consequently, it is not economically practicable, often not even possible, to shape the member for a close fit to its stress

Note.—Written comments are invited for publication; the last discussion should be submitted by July 1, 1951.

[1] Head Designing Engr., Bureau of Yards and Docks, U. S. Navy Dept., Washington, D. C.

pattern. As a result, material could not be used efficiently in conventional arrangements of framing. This fact, in turn, suggests a re-examination of present concepts and practice of structural design.

The problem of stress control (making the stress pattern as favorable as possible for proportioning the several sections) is not as hopeless as might first appear. A possible solution is already suggested by the flexural behavior of a member in a continuous framing. When the sections of the member, originally having a constant cross section along the span, are increased near its supports, the moment ordinates under a given lateral loading will increase correspondingly toward the ends of the span and decrease at the middle. Furthermore, if the increase in the end sections is accompanied with reductions in section at the middle of the member, the resulting changes in the moment ordinates become still more pronounced. From these observations it is not difficult to conclude that, as the shape of the member is changed from its original outline of constant cross section to that of wedge, the bending moment diagram will change correspondingly from the initial irregular outline of reversed flexure to the familiar pattern of single flexure—as obtained from two simple cantilevers.

The concept of wedge-beam framing is predicated on this consideration. It provides a method of establishing a consistent stress pattern to which the framing is closely fitted. The resulting arrangement assures more advantageous use of the materials of framing and thus yields appreciable savings in weight and economies in the cost of construction.

PART I. FRAMING

In general, wedge-beam framing may be defined as a system composed of wedge-shaped members arranged to provide a consistent or controllable stress

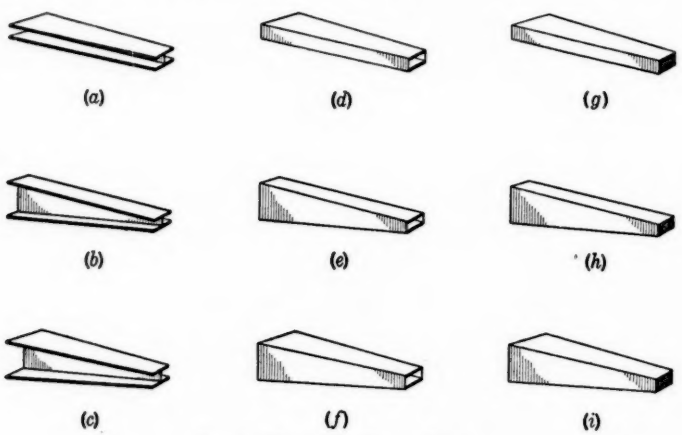


FIG. 1.—TYPES OF WEDGE BEAMS

pattern. For this purpose use is made of a hinged connection, or a joint of small moment capacity, at the tapered or narrow end of each member, to form a series of fixed inflection points in the flexural path of a framing under any given condition of loading. To assure the stability of the system, the connec-

tion at the butt or wide end of each member is made rigid. Thus the resulting arrangement is not fully continuous, as in rigid framing, nor is it like a simple framing where joints are assumed to have no moment resistance.

The Basic Element of Framing

A wedge beam is essentially a framing member of variable cross section. The variation may be due to either a single taper or a double taper, affecting

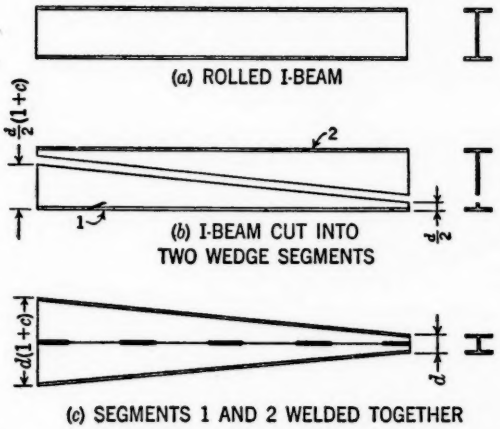


Fig. 2.—Method of Forming a Wedge Beam from an I-Beam

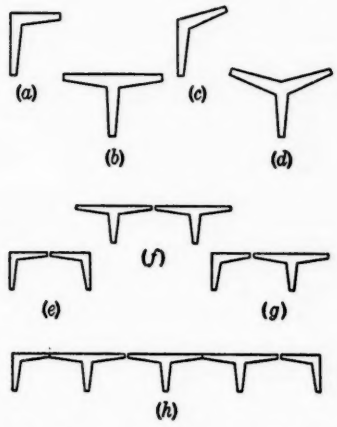


Fig. 3.—Subassemblies and Assemblies of Wedge-Beam Framing

the outlines of the web and the flanges of the beam. That is, a wedge beam may have either a tapering web or a tapering flange only, or it may have a tapering web and tapering flanges as well. In general, the taper will conform to a straight line; however, in some cases, a curved outline may also be used. In cross section, wedge beams may have the same shapes as those normally utilized in structural framing—namely, an I-section, a channel section, and a rectangle of either solid or hollow section. Some of these shapes and outlines are shown in Fig. 1.

A wedge beam may be constructed of steel, concrete, or timber. In the case of steel, any desired shape and outline can readily be obtained by welded plating. Rolled steel I-beam sections can also be used in building a wedge of constant width and of tapering depth, as indicated in Fig. 2. For concrete construction, hollow and solid rectangular sections can be utilized in obtaining the desired wedge, the former section being particularly adaptable to precast work.

Arrangement of Framing

Components of framing and the manner of assembly are shown diagrammatically in Fig. 3. For reference purposes, the main assembly may be considered to be composed of a series of statically determinate frames called "simple frames." A simple frame, in turn, is composed of one or two subassemblies each of which is formed by two or three wedge members rigidly joined at their butt ends. A two-member subassembly conforms to an inverted

L-outline, and is thus called an "L-segment." The arm of an L-segment may extend either horizontally, as in Fig. 3(*a*), or may be placed on an upward slope as in Fig. 3(*c*). A three-member subassembly has the outline of either a T (Fig. 3(*b*)), or a Y (Fig. 3(*d*)) and is, accordingly, called a "T-segment," or a "Y-segment." Figs. 3(*e*), 3(*f*), and 3(*g*) are three types of simple frames, composed, respectively, of two L-segments, two T-segments, and a combination of an L-segment and a T-segment. Fig. 3(*h*) shows a framing assembly consisting of a T-segment linked to a simple frame at each side.

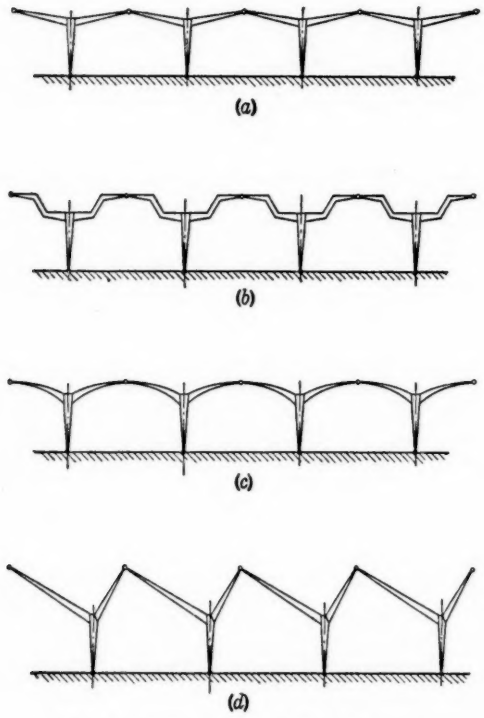


FIG. 4.—TYPES OF WEDGE-BEAM FRAMING

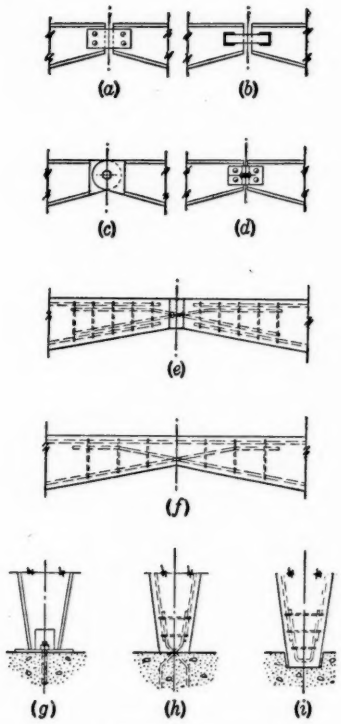


FIG. 5.—TYPES OF CONNECTIONS

In general, the wedge members are designed to provide a symmetrical arrangement in each bay of the framing. However, in certain cases it may be necessary to use an unsymmetrical arrangement—in elevation as well as in cross section—to conform to a given building outline (such as the unsymmetrical gable framing shown in Fig. 4(*d*)) or to obtain a more economical design.

In elevational outline, wedge-beam framing is adaptable also to buildings where the roof may have an outline differing from that of a flat or sloping roof. Two examples of this case are shown in Figs. 4(*b*) and 4(*c*).

CONNECTIONS

As stated in the preceding section, the members in a wedge-beam framing are connected by a rigid joint at the butt end and a hinge at the tapered end of

each wedge. Theoretically, the hinged connections should have no moment resistance since their use is intended to provide fixed inflection points in the stress pattern of a frame. In certain types of framing, it will be feasible to design a connection that will nearly satisfy this condition of restraint. However, if the detail for such a joint should entail much expense, then use may be made of a so-called flexible connection or a joint of limited rigidity. A joint of this type will usually yield and rotate when its small moment resisting capacity is overcome. Some suggested details conforming to this concept are given in Fig. 5. Details (*a*) and (*d*), Fig. 5, utilize riveted or welded connection gussets and clip angles, respectively. Detail (*b*) consists of a welded link bar of narrow width. Detail (*c*) is a simplified hinge connection, consisting of two overlapping end plates and an ordinary bolt. Detail (*g*) illustrates a simple anchor-bolt connection suitable for use at the bases of steel columns. Details (*e*), (*f*), (*h*), and (*i*), Fig. 5, are applicable in concrete construction. Of these, details (*e*) and (*h*) are similar to the semiflexible hinge of A. Mesnager.[2] Detail (*f*) has the advantage of a simplified construction but it is also subject to cracking; and it may be used where appearance is not of prime importance. Detail (*i*), Fig. 5, is suitable for use at column bases where no net uplift is present.

Range of Application

Wedge-beam framing can be utilized in a variety of building frames of steel, concrete, or timber. It is adaptable to riveted and conventionally poured-in-place concrete construction and, more particularly, to welded-steel and precast concrete construction. It can be used in tier-type, multistory frames and, still more advantageously, in single-story industrial buildings. These applications will result in large reductions in the weight of the framing materials, as well as in appreciable savings in construction costs. In addition, wedge-beam framing provides a framing outline of pleasing appearance.

PART II. ANALYSIS

A simple frame, as defined in Part I and sketched in Figs. 3(*e*), 3(*f*), and 3(*g*), is a statically determinate, two-legged, three-hinged bent. However, when two or more simple frames are linked together to form an assembly, the resulting arrangement becomes statically indeterminate. Accordingly, the analysis of a wedge-beam framing of two or more bays falls in the category of statically indeterminate structures. However, in comparison to rigid framing, the number of unknowns in a wedge-beam framing is relatively small. These unknowns consist of the two hinge reactions at each joint of linkage of adjacent simple frames. To determine the values of these hinge reactions, it is first necessary to establish their interrelation. This is obtained from the simple consideration that two adjacent simple frames will have the same deflections at a common link. The next step then is to solve the resulting series of simultaneous equations. More specifically, the procedure to be followed in the analysis will consist of five steps, as follows:

[2] "Design of Hinges and Articulations in Reinforced Concrete Structures," by George C. Ernst, *Transactions*, ASCE, Vol. 106, 1941, p. 862.

1. Subdivide the main assembly or framing into its component simple frames, and indicate the two unknown hinge reactions at each joint of interconnection or linkage. This step will result in a series of statically determinate simple frames, each frame subjected to unknown H-forces and V-forces on each side, in addition to its own loading.

2. Draw the equilibrium or simple moment diagram of each simple frame due to each unknown H-force and V-force, as well as one under the given loading of the frame. This will result, in general, in five diagrams.

3. Using the moment diagrams in step 2, write an expression for the horizontal and vertical deflection at each joint of linkage of each simple frame. In general, four equations per simple frame will result.

4. Equate the deflections of adjacent simple frames. By so equating a set of simultaneous equations, the number of which will correspond to the number of the unknown hinge reactions, results.

5. From the equations in step 4, obtain the values of the unknown hinge reactions. This procedure will now be elaborated in its applications to various types of rectangular frames.

Detachment of Simple Frames and the Determination of the Number of Unknown Hinge Reactions

The subdivision or the cutting of an assembly into its component simple frames does not require special rules. In most cases, the separation is made in every other bay of an assembly, beginning in the second bay. For assemblies having an odd number of bays, such as in Fig. 6, the separation will yield $\left(\frac{n-1}{2}\right)$-cuts, n indicating the number of bays. If the loading is unsymmetrical, as in Figs. 6(d), 6(e), and 6(f), the number of unknown hinge reactions at the cut joints will become $n - 1$. If the loading and framing are symmetrical, Figs. 6(a), 6(b), and 6(c), the number of the unknown hinge reactions is reduced to $\frac{n-1}{2}$.

When the assembly is composed of an even number of bays and the loading is unsymmetrical, Figs. 7(d), 7(e), and 7(f), the severance of hinges in alternating bays will result in $n/2$ simple frames and one L-segment having two interdependent hinge reactions. Accordingly, the number of unknown hinge reactions will become $n - 1$. If the loading and the framing are symmetrical, the separations will be made as indicated in Figs. 7(a), 7(b), and 7(c). To maintain the symmetry of arrangement and thus reduce the number of the corresponding unknown hinge reactions to a minimum, the assembly is cut in two adjacent bays at the center to form a T-segment. The number of unknown hinge reactions for this group of assemblies is given by $n/2$.

In the case of multistory assemblies, each story may be considered as a separate frame and treated as outlined in the preceding paragraphs. The arrangement, as applied to a three-story frame, is shown in Fig. 8.

Deflection Equations

Development.—In general, two equations can be derived from the two deflections at each connecting joint of adjacent simple frames. As stated in the

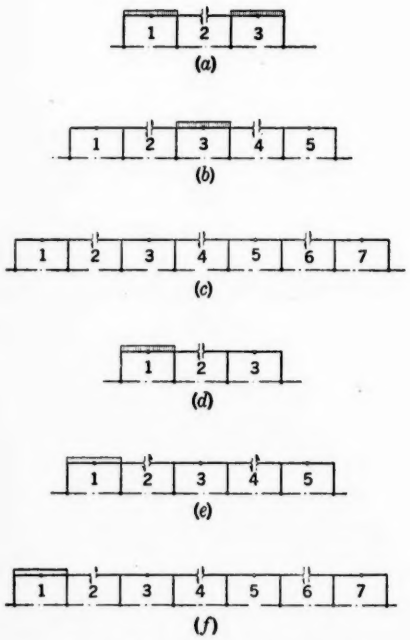


FIG. 6.—SUBDIVISION OF AN ODD NUMBER OF BAYS

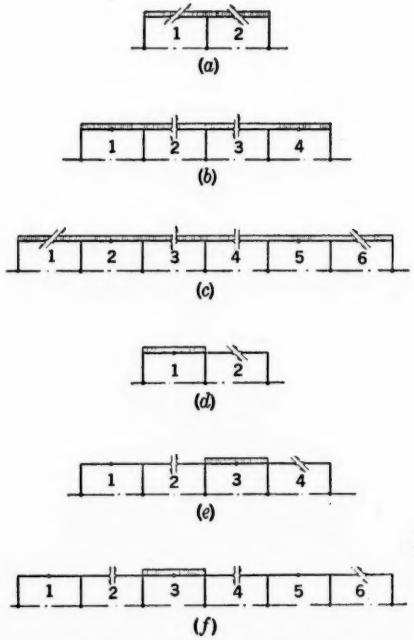


FIG. 7.—SUBDIVISION OF AN EVEN NUMBER OF BAYS

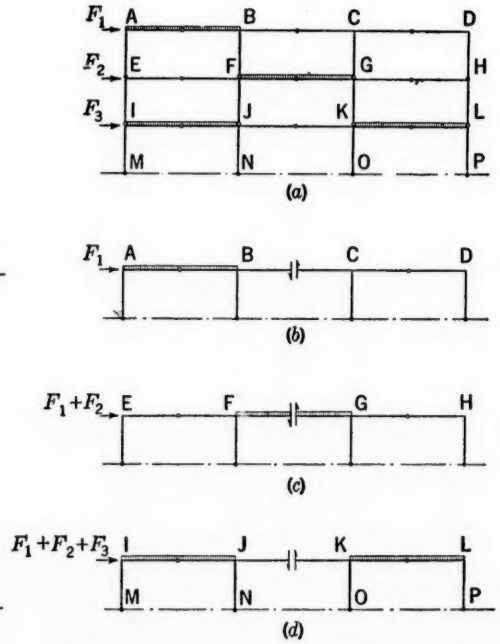


FIG. 8.—SUBDIVISION OF A MULTISTORY FRAME

preceding section, these are obtained by equating the horizontal and vertical deflections of the joint from one frame to the corresponding expressions in the other frame. The deflection of a simple frame is given by the familiar expression:

$$\Delta_D = \int \frac{M_s\, m_s\, \delta s}{E\, I_z} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

in which Δ_D is the deflection of a joint D, measured in the desired direction; M_s is the simple frame moment due to the applied loading; m_s is the simple frame moment due to a unit load applied at the joint in the desired direction of the deflection; δs is an element of length; E and I are the modulus of elasticity and the moment of inertia, respectively; and the summation includes all the members of the frame.

Since each member of the framing has a rigid joint at one end and a hinged connection at the other, the bending moment diagram of each member is similar to that of a cantilever beam. The moment may be due to an end force, such as the hinge reaction normal to the axis or end shear, or to a given loading applied between the two ends of a member. Let P_M represent the end force, or the normal hinge reaction, which produces the simple moment M_s in a member, and let p_m represent the end force producing a moment m_s. Then, the contribution of a member to the deflection of the simple frame, Eq. 1, may be considered as the deflection, Δ, of a cantilever subjected to a load $P_M\, p_m$ at its end:

$$\Delta = P_M\, p_m \frac{l^3}{E\, I} k \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

in which k is a constant depending on the shape of the member; l is the length; and I is the moment of inertia at the tapered end of the member. Formulas for k corresponding to typical loadings are given in Tables 1 to 5.

The deflection of a member due to a loading applied between the ends is the same as that of a simple cantilever. For concentrated load P, the expression is

$$\Delta_p = \frac{P\, l^3}{E\, I} k_p \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3a)$$

in which k_p is a constant depending on the shape of the member and the position of the load. Similarly, for a uniformly distributed loading w, the expression is

$$\Delta_w = \frac{w\, l^4}{E\, I} k_w \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3b)$$

k_w being the shape and the load constant for this case.

With this concept, to obtain the deflection δ_D of a simple frame, one need only sum up the deflections Δ of its component members, as given by Eqs. 2 and 3.

Special Case.—To illustrate the procedure, consider first the relatively simple cases of framing that contain only a few bays, such as: (*a*) The three-bay frame with symmetrical framing and loading; (*b*) the three-bay frame, with symmetrical framing and unsymmetrical loading; (*c*) the two-bay frame with unsymmetrical framing and loading; and (*d*) the four-bay frame with symmetrical framing and loading.

(*a*) Three-Bay Frame, Symmetrical Framing and Loading.—The frame in Fig. 9(*a*) illustrates this case. Because of the symmetry of framing and load-

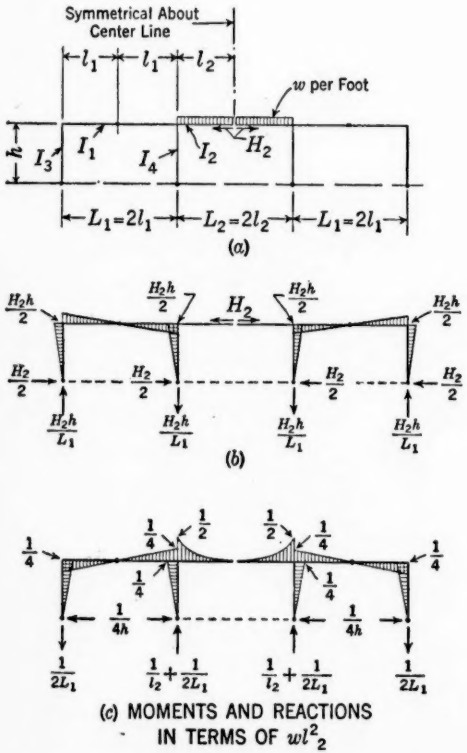


FIG. 9.—THREE-BAY FRAME WITH SYMMETRICAL FRAMING AND LOADING

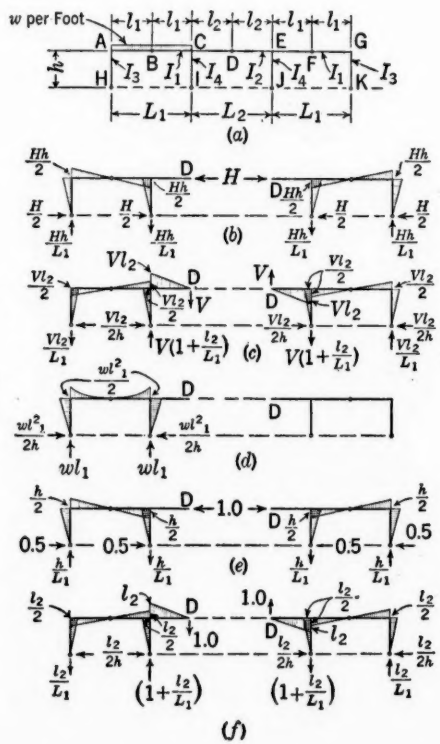


FIG. 10.—THREE-BAY FRAME WITH SYMMETRICAL FRAMING AND UNSYMMETRICAL LOADING

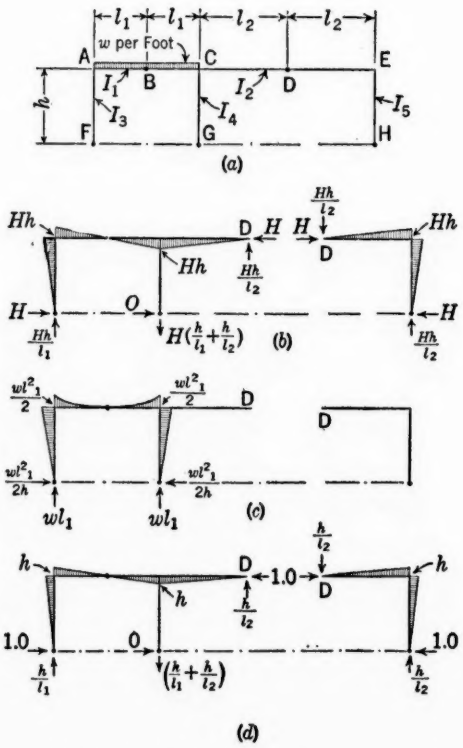
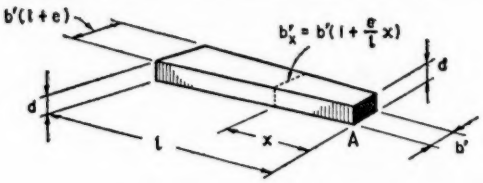


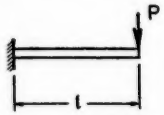
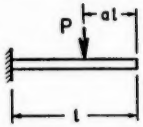
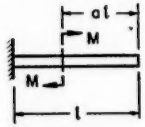
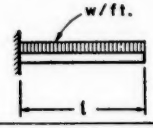
FIG. 11.—TWO-BAY FRAME WITH UNSYMMETRICAL FRAMING AND LOADING

TABLE 1.—DEFLECTION FORMULAS FOR TAPERED-FLANGE SOLID RECTANGULAR BEAMS

$I_x = I(1+\frac{e}{l}x)$



$I_A = \frac{1}{12}b'd^3 = I$

LOADING	DEFLECTION
	$\Delta_A = \frac{Pl^3}{EI}\cdot\frac{1}{e^2}\left[\frac{e}{2}-1+\frac{1}{e}\log_e(1+e)\right]$ $= \frac{Pl^3}{EI}\cdot k$
	$\Delta_A = \frac{Pl^3}{EI}\cdot\frac{1}{e^2}\left[a-1+\frac{e}{2}+ea(\frac{a}{2}-1)+(a+\frac{1}{e})\log_e\frac{1+e}{1+ea}\right]$ $= \frac{Pl^3}{EI}\cdot k_p$
	$\Delta_A = \frac{Ml^2}{EI}\cdot\frac{1}{e^2}\left[e(1-a)-\log_e\frac{1+e}{1+ea}\right]$ $= \frac{Ml^2}{EI}\cdot k_m$
	$\Delta_A = \frac{wl^4}{EI}\cdot\frac{1}{2e^3}\left[\frac{e(2e-3)+6}{6}-\frac{1}{e}\log_e(1+e)\right]$ $= \frac{wl^4}{EI}\cdot k_w$

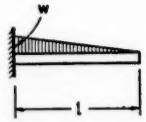
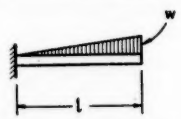
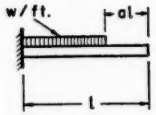
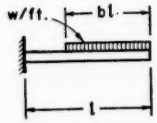
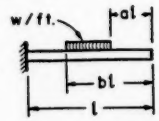
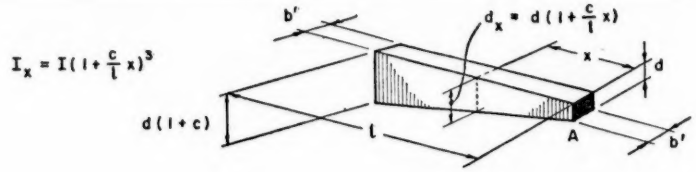
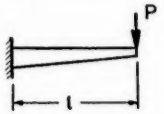
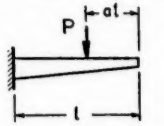
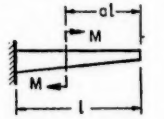
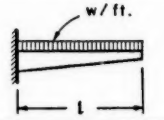
	$\Delta_A = \frac{wl^4}{EI} \cdot \frac{1}{6e^4}\left[\frac{e(3e^2-4e+6)-12}{12} + \frac{1}{e}\log_e(1+e)\right]$ $= \frac{wl^4}{EI} \cdot k_{w1}$
	$\Delta_A = \frac{wl^4}{EI} \cdot \frac{1}{6e^4}\left[\frac{e(9e^2-14e+30)+12}{12} - (3+\frac{1}{e})\log_e(1+e)\right]$ $= \frac{wl^4}{EI} \cdot k_{w2}$
	$\Delta_A = \frac{wl^4}{EI} \cdot \frac{1}{2e^4}\left\{-e(1+e)(1-a)(ea+3) + \frac{1+e}{6}\left[e(2e+13)+11\right] - \frac{1+ea}{6}\left[ea(2ea+13)+11\right] - (ea+1)^2\log_e\frac{1+e}{1+ea}\right\} = \frac{wl^4}{EI} \cdot k_{w3}$
	$\Delta_A = \frac{wl^4}{EI} \cdot \frac{1}{2e^4}\left\{-\frac{11}{6} - eb(1+e)\left[e(b-1)+3\right] + (1+eb)\left[(eb+2)^2-1\right] - \frac{(1+eb)^2}{6}(4eb+7) - (1+eb)^2\log_e(1+eb) + eb(eb+2)\log_e(1+e)\right\} = \frac{wl^4}{EI} \cdot k_{w4}$
	$\Delta_A = \frac{wl^4}{EI} \cdot \frac{1}{2e^4}\left\{-e(1+e)(b-a)\left[e(a+b-1)+3\right] - (1+ea)\left[(ea+2)^2-1\right] + (1+eb)\left[(eb+2)^2-1\right] + \frac{(1+ea)^2}{6}(4ea+7) - \frac{(1+eb)^2}{6}(4eb+7) + e\left[e(b^2-a^2)+2(b-a)\right]\log_e\frac{1+e}{1+eb} - (ea+1)^2\log_e\frac{1+eb}{1+ea}\right\} = \frac{wl^4}{EI} \cdot k_{w5}$

TABLE 2.—Deflection Formulas for Tapered-Web Solid Rectangular Beams



LOADING	DEFLECTION
	$\Delta_A = \frac{Pl^3}{EI}\cdot\frac{1}{c^2}\left[-\frac{2+3c}{2(1+c)^2}+\frac{1}{c}\log_e(1+c)\right]$ $= \frac{Pl^3}{EI}\cdot k$
	$\Delta_A = \frac{Pl^3}{EI}\cdot\frac{1}{c^3}\left[\frac{2+ca}{1+c}-\frac{3+2ca}{2(1+ca)}-\frac{1+ca}{2(1+c)^2}+\log_e\frac{1+c}{1+ca}\right]$ $= \frac{Pl^3}{EI}\cdot k_P$
	$\Delta_A = \frac{Ml^2}{EI}\cdot\frac{1}{c^2}\left[-\frac{1}{1+c}+\frac{1}{1+ca}+\frac{1}{2(1+c)^2}-\frac{1}{2(1+ca)^2}\right]$ $= \frac{Ml^2}{EI}\cdot k_m$
	$\Delta_A = \frac{wl^4}{EI}\cdot\frac{1}{2c^3}\left[\frac{c(2c+9)+6}{2(1+c)^2}-\frac{3}{c}\log_e(1+c)\right]$ $= \frac{wl^4}{EI}\cdot k_w$

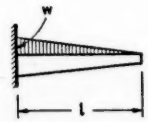
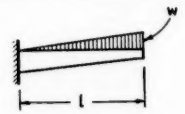
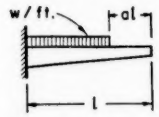
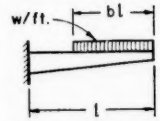
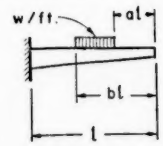
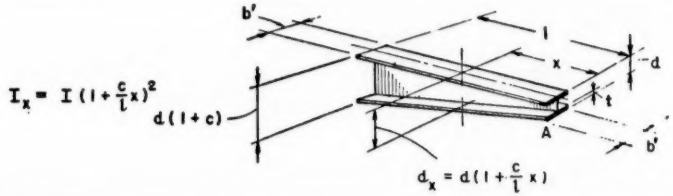
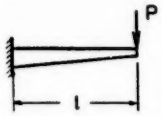
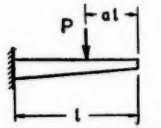
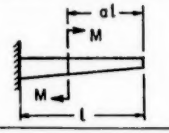
	$\Delta_A = \frac{wl^4}{EI} \cdot \frac{1}{6c^4}\left[\frac{c(c^2-4c-18)-12}{2(1+c)^2} + \frac{6}{c}\log_e(1+c)\right]$ $= \frac{wl^4}{EI} \cdot k_{w1}$
	$\Delta_A = \frac{wl^4}{EI} \cdot \frac{1}{6c^4}\left[\frac{c(5c^2+31c+36)+12}{2(1+c)^2} - (9+\frac{6}{c})\log_e(1+c)\right]$ $= \frac{wl^4}{EI} \cdot k_{w2}$
	$\Delta_A = \frac{wl^4}{EI} \cdot \frac{1}{2c^2}\left\{\frac{1-a}{c} - \left[a(a+\frac{4}{c}) + \frac{3}{c^2}\right]\left[\frac{1}{1+c} - \frac{1}{1+ca}\right] + \frac{(1+ca)^2}{2c^2}\left[\frac{1}{(1+c)^2} - \frac{1}{(1+ca)^2}\right] - \frac{1}{c}(2a+\frac{3}{c})\log_e\frac{1+c}{1+ca}\right\} = \frac{wl^4}{EI} \cdot k_{w3}$
	$\Delta_A = \frac{wl^4}{EI} \cdot \frac{1}{2c^2}\left\{\frac{b(4+cb)}{c(1+c)} - b(\frac{1}{c}+\frac{b}{2})\left[\frac{1}{(1+c)^2} - \frac{1}{(1+cb)^2}\right] - \frac{1}{2c^2}\left[1 - \frac{1}{(1+cb)^2}\right] - \frac{3}{c^2}\log_e(1+cb) + \frac{2b}{c}\log_e\frac{1+c}{1+cb}\right\} = \frac{wl^4}{EI} \cdot k_{w4}$
	$\Delta_A = \frac{wl^4}{EI} \cdot \frac{1}{2c^2}\left\{\frac{b-a}{c} + (b-a)(a+b+\frac{4}{c})\left[\frac{1}{1+c} - \frac{1}{1+cb}\right] + \left[a(a+\frac{4}{c}) + \frac{3}{c^2}\right]\left[\frac{1}{1+ca} - \frac{1}{1+cb}\right] - (b-a)(\frac{1}{c}+\frac{a+b}{2})\left[\frac{1}{(1+c)^2} - \frac{1}{(1+cb)^2}\right] - \frac{(1+ca)^2}{2c^2}\left[\frac{1}{(1+ca)^2} - \frac{1}{(1+cb)^2}\right] + \frac{2(b-a)}{c}\log_e\frac{1+c}{1+cb} - \frac{1}{c}(2a+\frac{3}{c})\log_e\frac{1+cb}{1+ca}\right\} = \frac{wl^4}{EI} \cdot k_{w5}$

TABLE 3.—DEFLECTION FORMULAS FOR TAPERED-WEB I-BEAMS



LOADING	DEFLECTION
	$\Delta_A = \frac{Pl^3}{EI}\cdot\frac{1}{c^2}\left[\frac{2+c}{1+c} - \frac{2}{c}\log_e(1+c)\right]$ $= \frac{Pl^3}{EI}\cdot k$
	$\Delta_A = \frac{Pl^3}{EI}\cdot\frac{1}{c^2}\left[\frac{(1-a)(2+c)}{1+c} - \frac{2+ca}{c}\log_e\frac{1+c}{1+ca}\right]$ $= \frac{Pl^3}{EI}\cdot k_p$
	$\Delta_A = \frac{Ml^2}{EI}\cdot\frac{1}{c^2}\left[\frac{1}{1+c} - \frac{1}{1+ca} + \log_e\frac{1+c}{1+ca}\right]$ $= \frac{Ml^2}{EI}\cdot k_m$

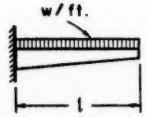
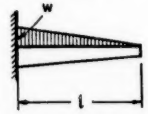
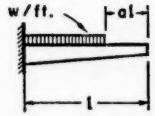
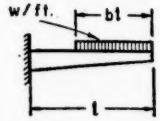
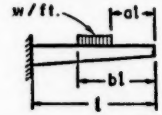
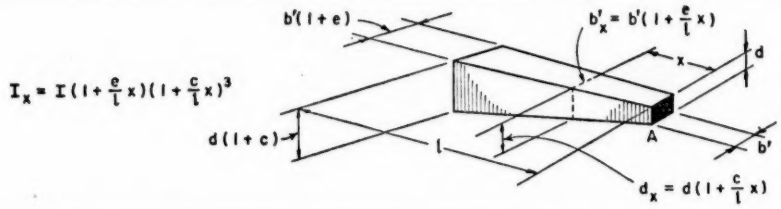
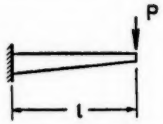
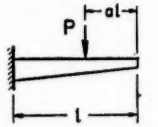
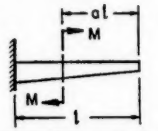
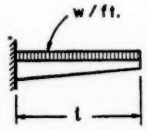
	$\Delta_A = \frac{wl^4}{EI} \cdot \frac{1}{2c^3}\left[\frac{c(c-3)-b}{2(1+c)} + \frac{3}{c}\log_e(1+c)\right] = \frac{wl^4}{EI} \cdot k_w$
	$\Delta_A = \frac{wl^4}{EI} \cdot \frac{1}{6c^4}\left[\frac{c(c^2-2c+6)+12}{3(1+c)} - \frac{4}{c}\log_e(1+c)\right] = \frac{wl^4}{EI} \cdot k_{w1}$
	$\Delta_A = \frac{wl^4}{EI} \cdot \frac{1}{6c^4}\left[\frac{c(7c^2-23c-66)-24}{6(1+c)} + (9+\frac{4}{c})\log_e(1+c)\right] = \frac{wl^4}{EI} \cdot k_{w2}$
	$\Delta_A = \frac{wl^4}{EI} \cdot \frac{1}{2c^2}\left\{(1-a)(\frac{1}{2}-\frac{1}{c}-\frac{3}{2}a) - \frac{1-a}{1+c}(a+1+\frac{2}{c}) + \left[a(a+\frac{4}{c})+\frac{3}{c^2}\right]\log_e\frac{1+c}{1+ca}\right\} = \frac{wl^4}{EI} \cdot k_{w3}$
	$\Delta_A = \frac{wl^4}{EI} \cdot \frac{1}{2c^2}\left[b(2-\frac{1}{c}-\frac{3b}{2}) - \frac{b}{1+c}(b+\frac{2}{c}) + \frac{3}{c^2}\log_e(1+cb) - b(b+\frac{4}{c})\log_e\frac{1+c}{1+cb}\right] = \frac{wl^4}{EI} \cdot k_{w4}$
	$\Delta_A = \frac{wl^4}{EI} \cdot \frac{1}{2c^2}\left\{(b-a)\left[2-\frac{1}{c}-\frac{3}{2}(a+b)\right] - \frac{b-a}{1+c}(a+b+\frac{c}{2}) + \left[a(a+\frac{4}{c})+\frac{3}{c^2}\right]\log_e\frac{1+cb}{1+ca} - (b-a)(a+b+\frac{4}{c})\log_e\frac{1+c}{1+cb}\right\} = \frac{wl^4}{EI} \cdot k_{w5}$

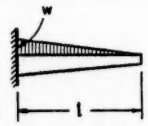
TABLE 4.—Deflection Formulas for Tapered-Flange and Tapered-Web Solid Rectangular Beams



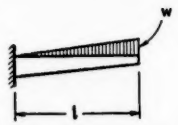
LOADING	DEFLECTION
	$\Delta_A = \frac{Pl^3}{EI}\cdot\left[\frac{e-2c}{c(e-c)^2(1+c)} - \frac{2+c}{2c(e-c)(1+c)^2} + \frac{1}{(e-c)^3}\log_e(1+e) - \frac{1}{(e-c)^3}\log_e(1+c)\right] = \frac{Pl^3}{EI}\cdot k$
	$\Delta_A = \frac{Pl^3}{EI}\cdot\left\{\frac{c(ca+2)-e}{c^2(e-c)^2}\left[\frac{1}{1+c}-\frac{1}{1+ca}\right] + \frac{1+ca}{2c^2(e-c)}\left[\frac{1}{(1+c)^2}-\frac{1}{(1+ca)^2}\right] + \frac{1+ea}{(e-c)^3}\log_e\frac{1+e}{1+ea} - \frac{1+ea}{(e-c)^3}\log_e\frac{1+c}{1+ca}\right\} = \frac{Pl^3}{EI}\cdot k_P$
	$\Delta_A = \frac{Ml^2}{EI}\cdot\left\{-\frac{1}{(e-c)^2}\left[\frac{1}{1+c}-\frac{1}{1+ca}\right] - \frac{1}{2c(e-c)}\left[\frac{1}{(1+c)^2}-\frac{1}{(1+ca)^2}\right] - \frac{e}{(e-c)^3}\log_e\frac{1+e}{1+ea} + \frac{e}{(e-c)^3}\log_e\frac{1+c}{1+ca}\right\} = \frac{Ml^2}{EI}\cdot k_m$



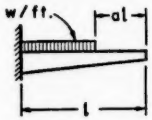
$$\Delta_A = \frac{wl^4}{EI} \cdot \frac{1}{2}\left[\frac{3c-2e}{c^2(e-c)^2(1+c)} + \frac{2+c}{2c^2(e-c)(1+c)^2} - \frac{1}{e(e-c)^3}\log_e(1+e) + \frac{e^2-3c(e-c)}{c^3(e-c)^3}\log_e(1+c)\right] = \frac{wl^4}{EI} \cdot k_w$$



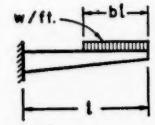
$$\Delta_A = \frac{wl^5}{EI} \cdot \frac{1}{6}\left[\frac{3e-4c}{c^3(e-c)^2(1+c)} - \frac{2+c}{2c^3(e-c)(1+c)^2} + \frac{1}{e^2(e-c)^3}\log_e(1+e) - \frac{6c^2-8ec+3e^2}{c^4(e-c)^3}\log_e(1+c) + \frac{1}{ec^3}\right] = \frac{wl^5}{EI} \cdot k_{w1}$$



$$\Delta_A = \frac{wl^5}{EI} \cdot \left[k_w - k_{w1}\right] = \frac{wl^5}{EI} \cdot k_{w2}$$

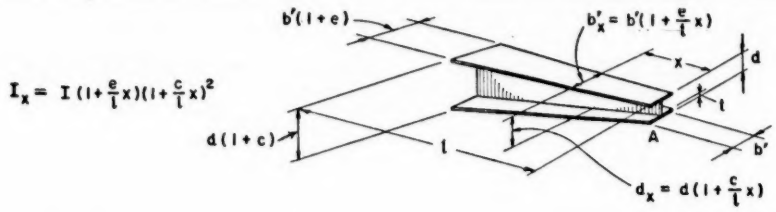


$$\Delta_A = \frac{wl^4}{EI} \cdot \frac{1}{2}\left\{-\frac{3c-2e+2ca(2c-e)+c^3a^2}{c^3(e-c)^2}\left[\frac{1}{1+c}-\frac{1}{1+ca}\right] - \frac{(1+ca)^2}{2c^3(e-c)}\left[\frac{1}{(1+c)^2}-\frac{1}{(1+ca)^2}\right] - \frac{(1+ea)^2}{e(e-c)^3}\log_e\frac{1+e}{1+ea} + \frac{e^2-3c(e-c)+ac^3(2+ea)}{c^3(e-c)^3}\log_e\frac{1+c}{1+ca}\right\} = \frac{wl^4}{EI} \cdot k_{w3}$$



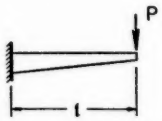
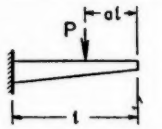
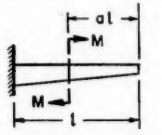
$$\Delta_A = \frac{wl^4}{EI} \cdot \left[k_w - k_{w3},\ a=b\right] = \frac{wl^4}{EI} \cdot k_{w4}$$

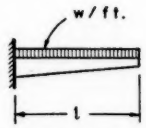
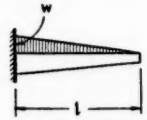
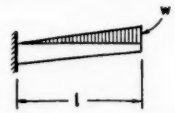
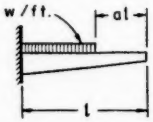
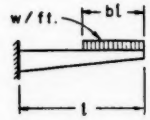
TABLE 5.—DEFLECTION FORMULAS FOR TAPERED-FLANGE AND TAPERED-WEB I-BEAMS



$I_x = I\left(1+\frac{e}{l}x\right)\left(1+\frac{c}{l}x\right)^2$

$I_A = \frac{b'td^2}{2} = I$ (approx.)

LOADING	DEFLECTION
	$\Delta_A = \frac{Pl^3}{EI}\cdot\left[\frac{-1}{c(e-c)(1+c)} + \frac{1}{e(e-c)^2}\log_e(1+e) - \frac{2c-e}{c^2(e-c)^2}\log_e(1+c)\right]$ $= \frac{Pl^3}{EI}\cdot k$
	$\Delta_A = \frac{Pl^3}{EI}\cdot\left\{\frac{(1+ca)}{c^2(e-c)}\left[\frac{1}{1+c}-\frac{1}{1+ca}\right] + \frac{1+ea}{e(e-c)^2}\log_e\frac{1+e}{1+ea} + \frac{e-c(2+ca)}{c^2(e-c)^2}\log_e\frac{1+c}{1+ca}\right\}$ $= \frac{Pl^3}{EI}\cdot k_P$
	$\Delta_A = \frac{Ml^2}{EI}\cdot\left\{\frac{-1}{c(e-c)}\left[\frac{1}{1+c}-\frac{1}{1+ca}\right] - \frac{1}{(e-c)^2}\log_e\frac{1+e}{1+ea} + \frac{1}{(e-c)^2}\log_e\frac{1+c}{1+ca}\right\}$ $= \frac{Ml^2}{EI}\cdot k_m$

	$\Delta_A = \frac{wl^4}{EI} \cdot \frac{1}{2}\left[\frac{1}{ec^2} + \frac{1}{c^2(e-c)(1+c)} - \frac{1}{e^2(e-c)^2}\log_e(1+e) + \frac{3c-2e}{c^3(e-c)^2}\log_e(1+c)\right]$ $= \frac{wl^4}{EI} \cdot k_w$
	$\Delta_A = \frac{wl^4}{EI} \cdot \frac{1}{6}\left[\frac{-1}{c^3(e-c)(1+c)} + \frac{c(e-2)-4e}{2e^2c^3} + \frac{1}{e^3(e-c)^2}\log_e(1+e) - \frac{4c-3e}{c^4(e-c)^2}\log_e(1+c)\right] = \frac{wl^4}{EI} \cdot k_{w1}$
	$\Delta_A = \frac{wl^4}{EI} \cdot \frac{1}{6}\left[\frac{3c+1}{c^3(e-c)(1+c)} + \frac{c(5e+2)+4e}{2e^2c^3} - \frac{3e+1}{e^3(e-c)^2}\log_e(1+e) + \frac{3c(3c-2e)+(4c-3e)}{c^4(e-c)^2}\log_e(1+c)\right] = \frac{wl^4}{EI} \cdot k_{w2}$
	$\Delta_A = \frac{wl^4}{EI} \cdot \frac{1}{2}\left\{\frac{1-a}{ec^2} - \frac{(1+ca)^2}{c^3(e-c)}\left[\frac{1}{1+c} - \frac{1}{1+ca}\right] - \frac{(1+ea)^2}{e^2(e-c)^2}\log_e\frac{1+e}{1+ea} + \frac{ca\left[c^2a+2(2c-e)\right]+3c-2e}{c^3(e-c)^2}\log_e\frac{1+c}{1+ca}\right\} = \frac{wl^4}{EI} \cdot k_{w3}$
	$\Delta_A = \frac{wl^4}{EI} \cdot \left[k_w - k_{w3}(a=b)\right]$ $= \frac{wl^4}{EI} \cdot k_{w4}$

ing, the vertical hinge reaction at the center of the middle bay beam is zero, and the horizontal hinge reaction, H_2, at the same location remains the only unknown force to be determined. This force may be obtained from the equation of horizontal deflection, Δ_{H2}, at the center hinge. For this purpose, the simple frame moment diagrams due to H_2 and w, are sketched as shown in Figs. 9(*b*) and 9(*c*). By substituting 1 for H_2, Fig. 9(*b*) will also give the simple frame moments due to a unit horizontal load at the center hinge. Then, to determine the contribution of the five members in one half of the frame to the deflection at the center hinge (by Eqs. 2 and 3):

For member 1—

$$\Delta_1 = 2\frac{H_2 h}{2 l_1}\frac{h}{2 l_1}\frac{I_1^3}{I_1}k_1 - 2\frac{w l_2^2}{4 l_1}\frac{h}{2 l_1}\frac{I_1^3}{I_1}k_1 \quad \ldots\ldots\ldots\ldots (4a)$$

for member 2—

$$\Delta_2 = 0. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4b)$$

for member 3—

$$\Delta_3 = \frac{H_2}{2}\frac{1}{2}\frac{h^3}{I_3}k_3 - \frac{w l_2^2}{4 h}\frac{1}{2}\frac{h^3}{I_3}k_3 \quad \ldots\ldots\ldots\ldots\ldots (4c)$$

and for member 4—

$$\Delta_4 = \frac{H_2}{2}\frac{1}{2}\frac{h^3}{I_4}k_4 + \frac{w l_2^2}{4 h}\frac{1}{2}\frac{h^3}{I_4}k_4 \quad \ldots\ldots\ldots\ldots\ldots (4d)$$

Since $\Delta_1 + \Delta_2 + \Delta_3 + \Delta_4 = \Delta_{H2} = 0$,

$$H_2 = \frac{w l_2^2}{2}\frac{2\frac{l_1}{h}\frac{k_1}{I_1} + \frac{k_3}{I_3} - \frac{k_4}{I_4}}{2 l_1\frac{k_1}{I_1} + h\left(\frac{k_3}{I_3} + \frac{k_4}{I_4}\right)} \quad \ldots\ldots\ldots\ldots\ldots (5)$$

The moment sign is governed by the direction of rotation of the forces about the rigid joint of a member. A clockwise rotation is plus (+) and a counterclockwise rotation is minus (−).

(*b*) Three-Bay Frame, Symmetrical Framing, Unsymmetrical Loading.—The frame in Fig. 10(*a*), illustrating this case, carries a uniformly distributed load in only the first bay. Because of the dissymmetry of loading, there are two unknown hinge reactions, H and V, at the center hinge D. The simple frame moment diagrams due to these two forces are shown in Figs. 10(*b*) and 10(*c*). The simple frame moments due to the loading in the first bay are as indicated in Fig. 10(*d*). Moments caused by a unit horizontal and vertical force at hinge D are given in Figs. 10(*e*) and 10(*f*), respectively. With these values, and applying Eqs. 2 and 3, the horizontal deflection at hinge D is

$$\Delta_{1H} = 4\frac{H h}{2 l_1}\frac{h}{2 l_1}\frac{I_1^3}{I_1}k_1 + V \times 0 + 0 \quad \ldots\ldots\ldots\ldots (6a)$$

$$\Delta_{2H} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6b)$$

$$\Delta_{3H} = 2\frac{H}{2}\frac{1}{2}\frac{h^3}{I_3}k_3 + V \times 0 + \frac{w l_1^2}{2 h}\frac{1}{2}\frac{h^3}{I_3}k_3 \quad \ldots\ldots\ldots\ldots (6c)$$

and

$$\Delta_{4H} = 2\frac{H}{2}\frac{1}{2}\frac{h^3}{I_4}k_4 + V \times 0 - \frac{w\,l^2_1}{2\,h}\frac{1}{2}\frac{h^3}{I_4}k_4 \quad \ldots\ldots (6d)$$

Since the deflection of the left side of the joint minus the deflection of the right side of joint equals zero,

$$\Delta_{1H} + \Delta_{2H} + \Delta_{3H} + \Delta_{4H} = \Delta_{HD} = 0 \quad \ldots\ldots (7)$$

and

$$H = \frac{w\,l_1}{2}\frac{\dfrac{k_4}{I_4} - \dfrac{k_3}{I_3}}{2\dfrac{k_1}{I_1} + \dfrac{h}{l_1}\left(\dfrac{k_3}{I_3} + \dfrac{k_4}{I_4}\right)} \quad \ldots\ldots (8)$$

Similarly, the vertical deflection at hinge D is

$$\Delta_{1V} = 4\frac{V\,l_2}{2\,l_1}\frac{l_2}{2\,l_1}\frac{l^3_1}{I_1}k_1 + H \times 0 + 0 \quad \ldots\ldots (9a)$$

$$\Delta_{2V} = 2\,V \times 1 \times \frac{l^3_1}{I_2}k_2 + H \times 0 \quad \ldots\ldots (9b)$$

$$\Delta_{3V} = 2\frac{V\,l_2}{2\,h}\frac{l_2}{2\,h}\frac{h^3}{I_3}k_3 + H \times 0 - \frac{w\,l^2_1}{2\,h}\frac{l_2}{2\,h}\frac{h^3}{I_3}k_3 \quad \ldots\ldots (9c)$$

and

$$\Delta_{4V} = 2\frac{V\,l_2}{2\,h}\frac{l_2}{2\,h}\frac{h^3}{I_4}k_4 + H \times 0 - \frac{w\,l^2_1}{2\,h}\frac{l_2}{2\,h}\frac{h^3}{I_4}k_4 \quad \ldots\ldots (9d)$$

As before, since $\Delta_{1V} + \Delta_{2V} + \Delta_{3V} + \Delta_{4V} = \Delta_{VD} = 0$,

$$V = \frac{w\,l_1}{2}\frac{\dfrac{k_3}{I_3} + \dfrac{k_4}{I_4}}{2\dfrac{l_2}{h}\left(\dfrac{k_1}{I_1} + 2\dfrac{l_2}{l_1}\dfrac{k_2}{I_2}\right) + \dfrac{l_2}{l_1}\left(\dfrac{k_3}{I_3} + \dfrac{k_4}{I_4}\right)} \quad \ldots\ldots (10)$$

(*c*) Two-Bay Frame, Unsymmetrical Framing and Loading.—The framing in Fig. 11(*a*) is composed of a simple frame and an L-segment. The two hinge reactions at the cut joint D being interrelated, there is only one unknown force (namely, H) that needs to be determined. Using the simple moment diagrams shown in Figs. 11(*b*), 11(*c*), and 11(*d*), and applying Eqs. 2 and 3,

$$\Delta_{1D} = 2\frac{H\,h}{l_1}\frac{h}{l_1}\frac{l^3_1}{I_1}k_1 + 0 \quad \ldots\ldots (11a)$$

$$\Delta_{2D} = 2\frac{H\,h}{l_2}\frac{h}{l_2}\frac{l^3_2}{I_2}k_2 \quad \ldots\ldots (11b)$$

$$\Delta_{3D} = H \times 1 \times \frac{h^3}{I_3}k_3 + \frac{w\,l^2_1}{2\,h}1\frac{h^3}{I_3}k_3 \quad \ldots\ldots (11c)$$

$$\Delta_{4D} = 0 \quad \ldots\ldots (11d)$$

and

$$\Delta_{5D} = H \times 1 \times \frac{h^3}{I_5}k_5 \quad \ldots\ldots (11e)$$

Since $\Delta_{1D} + \Delta_{2D} + \Delta_{3D} + \Delta_{4D} + \Delta_{5D} = \Delta_{DD} = 0$,

$$H = -\frac{w\,l_1}{2}\,\frac{\dfrac{k_3}{I_3}}{2\dfrac{k_1}{I_1} + 2\dfrac{l_2}{l_1}\dfrac{k_2}{I_2} + \dfrac{h}{l_1}\left(\dfrac{k_3}{I_3} + \dfrac{k_5}{I_5}\right)} \qquad (12)$$

(*d*) Four-Bay Frame, Symmetrical Framing and Loading.—The framing in Fig. 12(*a*) illustrating this case is composed of two simple frames and a T-segment. There are only two unknown hinge reactions (namely, H and V) at joint D. Their solution is obtained from horizontal and vertical deflection equations at the same joint, using the simple moment diagrams in Figs. 12(*b*), 12(*c*), and 12(*d*). Formulas for the horizontal deflection at hinge D are

$$\Delta_{1H} = 2\,\frac{H\,h}{2\,l_1}\,\frac{h}{2\,l_1}\,\frac{l^3_1}{I_1}\,k_1 - 2\,\frac{V\,l_2}{2\,l_1}\,\frac{h}{2\,l_1}\,\frac{l^3_1}{I_1}\,k_1 - \frac{2\,w\,l^2_2}{4}\,\frac{h}{2\,l_1}\,\frac{l^3_1}{I_1}\,k_1 \qquad (13a)$$

$$\Delta_{2H} = 0 \qquad (13b)$$

$$\Delta_{3H} = \frac{H}{2}\,\frac{1}{2}\,\frac{h^3}{I_3}\,k_3 - \frac{V\,l_2}{2\,h}\,\frac{1}{2}\,\frac{h^3}{I_3}\,k_3 + \frac{w}{2\,h}\left(l^2_1 - \frac{l^2_2}{2}\right)\frac{1}{2}\,\frac{h^3}{I_3}\,k_3 \qquad (13c)$$

$$\Delta_{4H} = \frac{H}{2}\,\frac{1}{2}\,\frac{h^3}{I_4}\,k_4 + \frac{V\,l_2}{2\,h}\,\frac{1}{2}\,\frac{h^3}{I_4}\,k_4 - \frac{w}{2\,h}\left(l^2_1 - \frac{l^2_2}{2}\right)\frac{1}{2}\,\frac{h^3}{I_4}\,k_4 \qquad (13d)$$

and

$$\Delta_{5H} = 0 \qquad (13e)$$

Since $\Delta_{1H} + \Delta_{2H} + \Delta_{3H} + \Delta_{4H} + \Delta_{5H} = \Delta_{HD} = 0$, the values of H and V at the hinge are found by combining Eqs. 13. Likewise, formulas for the vertical deflection at hinge D are

$$\Delta_{1V} = 2\,V\,\frac{l_2}{2\,l_1}\,\frac{l_2}{2\,l_1}\,\frac{l^3_1}{I_1}\,k_1 - 2\,\frac{H\,h}{2\,l_1}\,\frac{l_2}{2\,l_1}\,\frac{l^3_1}{I_1}\,k_1 + \frac{2\,w\,l^2_2}{4}\,\frac{l_2}{2\,l_1}\,\frac{l^3_1}{I_1}\,k_1 \qquad (14a)$$

$$\Delta_{2V} = 3\,V\,1\,\frac{l^3_2}{I_2}\,k_2 - \frac{w\,l^2_2}{2}\,\frac{l^3_2}{I_2}\,k_{2w} \qquad (14b)$$

$$\Delta_{3V} = V\,\frac{l_2}{2\,h}\,\frac{l_2}{2\,h}\,\frac{h^3}{I_3}\,k_3 - \frac{H}{2}\,\frac{l_2}{2\,h}\,\frac{h^3}{I_3}\,k_3 - \frac{w}{2\,h}\left(l^2_1 - \frac{l^2_2}{2}\right)\frac{l_2}{2\,h}\,\frac{l_2\,h^3}{I_3}\,k_3 \qquad (14c)$$

$$\Delta_{4V} = V\,\frac{l_2}{2\,h}\,\frac{l_2}{2\,h}\,\frac{h^3}{I_4}\,k_4 + \frac{H}{2}\,\frac{l_2}{2\,h}\,\frac{h^3}{I_4}\,k_4 - \frac{w}{2\,h}\left(l^2_1 - \frac{l^2_2}{2}\right)\frac{l_2}{2\,h}\,\frac{l_2\,h^3}{I_4}\,k_4 \qquad (14d)$$

and

$$\Delta_{5V} = 0 \qquad (14e)$$

With $\Delta_{1V} + \Delta_{2V} + \Delta_{3V} + \Delta_{4V} + \Delta_{5V} = \Delta_{VD} = 0$, the values of H and V at the hinge are found by combining Eqs. 14.

General Case.—The more general case is represented by the deflection equations at a hinge linking two simple frames in the interior bays of a multibay assembly. The bent in Fig. 13(*a*) is used to illustrate this case. In writing

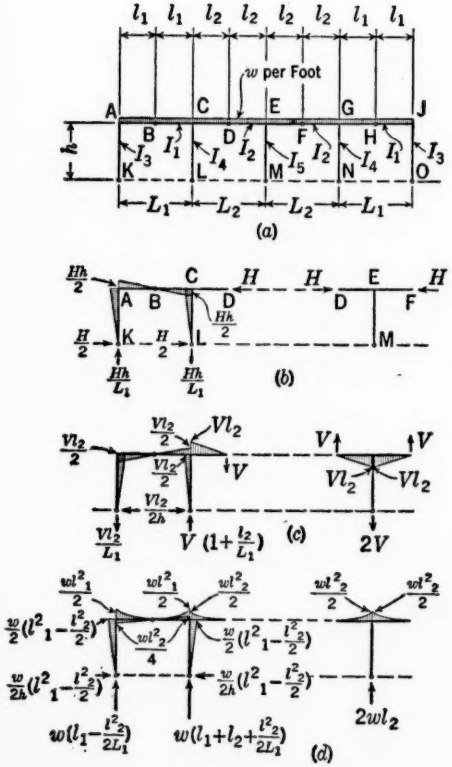


FIG. 12.—FOUR-BAY FRAME WITH SYMMETRICAL FRAMING AND LOADING

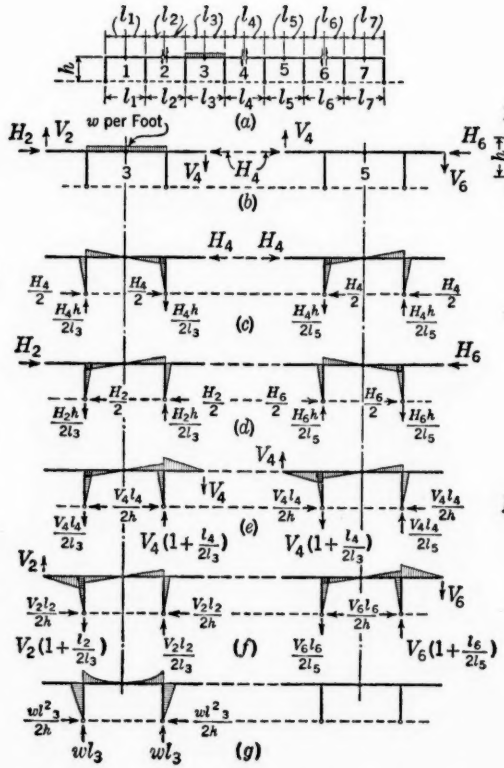


FIG. 13.—GENERAL CASE, THE MULTIBAY ASSEMBLY

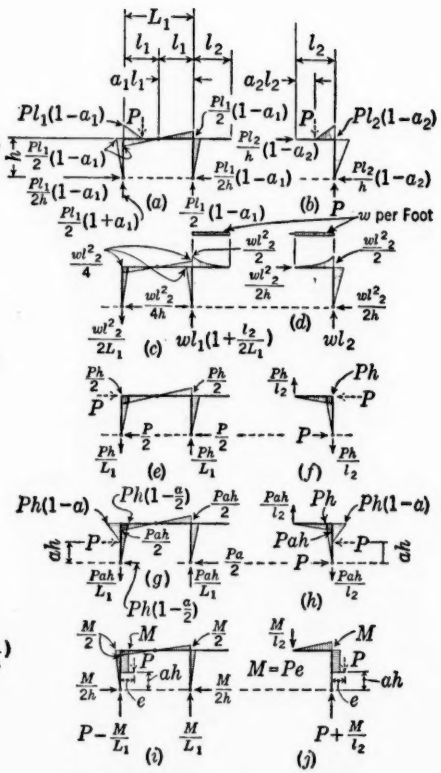
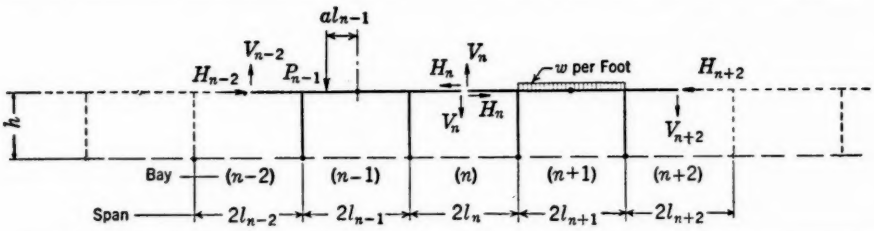


FIG. 14.—SIMPLE MOMENT DIAGRAMS, GENERAL CASES

the deflection equations at the hinge in the fourth or connecting bay, the effects of the six unknown hinge reactions, and the given loading in the third bay, must be considered as indicated in Fig. 13(*b*). The corresponding simple moment diagrams are shown in Figs. 13(*c*) to 13(*g*). To obtain the horizontal deflection, Δ_{H4}, it will be necessary to combine the moment ordinates of these

TABLE 6.—CONTRIBUTION OF MEMBERS TO THE DEFLECTION AT A GIVEN JOINT *n*



Member	C[a]	$h\,H_n$	$l_n\,V_n$	$h\,H_{n-2}$	$l_{n-2}\,V_{n-2}$	$h\,H_{n+2}$	$l_{n+2}\,V_{n+2}$	$w_{n+1}\,l^2_{n+1}$	$(1-a)\,l_{n-1}\,t_{n-1}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
(*a*) COEFFICIENTS FOR THE HORIZONTAL DEFLECTION Δ_{Hn} AT JOINT n									
Beam:									
$(n-1)_{L,R}$	C_1	$r_{n(n)1}$	$s_{n(n)1}$	$r_{n(n-2)1}$	$s_{n(n-2)1}$				$t_{n(n-1)1}$
$n_{L,R}$	C_2								
$(n+1)_{L,R}$	C_3	$r_{n(n)3}$	$s_{n(n)3}$			$r_{n(n+2)3}$	$s_{n(n+2)3}$		
Column:									
$(n-2)-(n-1)$	C_4	$r_{n(n)4}$	$s_{n(n)4}$	$r_{n(n-2)4}$	$s_{n(n-2)4}$				$t_{n(n-1)4}$
$(n-1)-n$	C_5	$r_{n(n)5}$	$s_{n(n)5}$	$r_{n(n-2)5}$	$s_{n(n-2)5}$				$t_{n(n-1)5}$
$n-(n+1)$	C_6	$r_{n(n)6}$	$s_{n(n)6}$			$r_{n(n+2)6}$	$s_{n(n+2)6}$	$t_{nw(n+1)6}$	
$(n+1)-(n+2)$	C_7	$r_{n(n)7}$	$s_{n(n)7}$			$r_{n(n+2)7}$	$s_{n(n+2)7}$	$t_{nw(n+1)7}$	
(*b*) COEFFICIENTS FOR THE VERTICAL DEFLECTION Δ_{Vn} AT JOINT n									
Beam:									
$(n-1)_{L,R}$	C'_1	$r'_{n(n)1}$	$s'_{n(n)1}$	$r'_{n(n-2)1}$	$s'_{n(n-2)1}$				$t'_{n(n-1)1}$
$n_{L,R}$	C'_2		$s'_{n(n)2}$						
$(n+1)_{L,R}$	C'_3	$r'_{n(n)3}$	$s'_{n(n)3}$			$r'_{n(n+2)3}$	$s'_{n(n+2)3}$		
Column:									
$(n-2)-(n-1)$	C'_4	$r'_{n(n)4}$	$s'_{n(n)4}$	$r'_{n(n-2)4}$	$s'_{n(n-2)4}$				$t'_{n(n-1)4}$
$(n-1)-n$	C'_5	$r'_{n(n)5}$	$s'_{n(n)5}$	$r'_{n(n-2)5}$	$s'_{n(n-2)5}$				$t'_{n(n-1)5}$
$n-(n+1)$	C'_6	$r'_{n(n)6}$	$s'_{n(n)6}$			$r'_{n(n+2)6}$	$s'_{n(n+2)6}$	$t'_{nw(n+1)6}$	
$(n+1)-(n+2)$	C'_7	$r'_{n(n)7}$	$s'_{n(n)7}$			$r'_{n(n+2)7}$	$s'_{n(n+2)7}$	$t'_{nw(n+1)7}$	

[a] $C = k\,h\,\frac{l}{I}$; $\quad C' = k\,l_n\,\frac{l}{I}$.

diagrams with those resulting from unit horizontal force at hinge 4. As a guide in setting up the desired deflection equation, the contribution of each member to the deflection at the joint under study is given in Table 6. Symbols C in Col. 2 denote the quantities—

$$C = k\,h\,\frac{l}{I} \qquad (15a)$$

—when applied to horizontal deflections (Table 6(a)), and—

$$C' = k\, l_n \frac{l}{I} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (15b)$$

—when applied to vertical deflections (Table 6(b)). The terms r, s, and t in each row designated by a member, when multiplied by the coefficient C and the respective column heading, will represent the contribution to deflection by that member. The sum of these contributions, when equated to zero, will result in the typical deflection expression containing the six unknown hinge reactions. The subscripts L and R in Col. 1 refer to the left and right segments respectively, of horizontal members, the two segments in each bay being combined into a single item. The expression for horizontal deflection (Table 6(a)) may be written in general form:

$$\Delta_{Hn} = \alpha_n H_n + \beta_n V_n + \alpha_{n-2} H_{n-2} + \beta_{n-2} V_{n-2} + \alpha_{n+2} H_{n+2} + \beta_{n+2} V_{n+2} + Q_n = 0 \ldots (16a)$$

Similarly, from the data in Table 6(b) for vertical deflection at joint n,

$$\Delta_{Vn} = \beta'_n V_n + \alpha'_n H_n + \beta'_{n-2} V_{n-2} + \alpha'_{n-2} H_{n-2} + \beta'_{n+2} V_{n+2} + \alpha'_{n+2} H_{n+2} + Q'_n = 0 \ldots (16b)$$

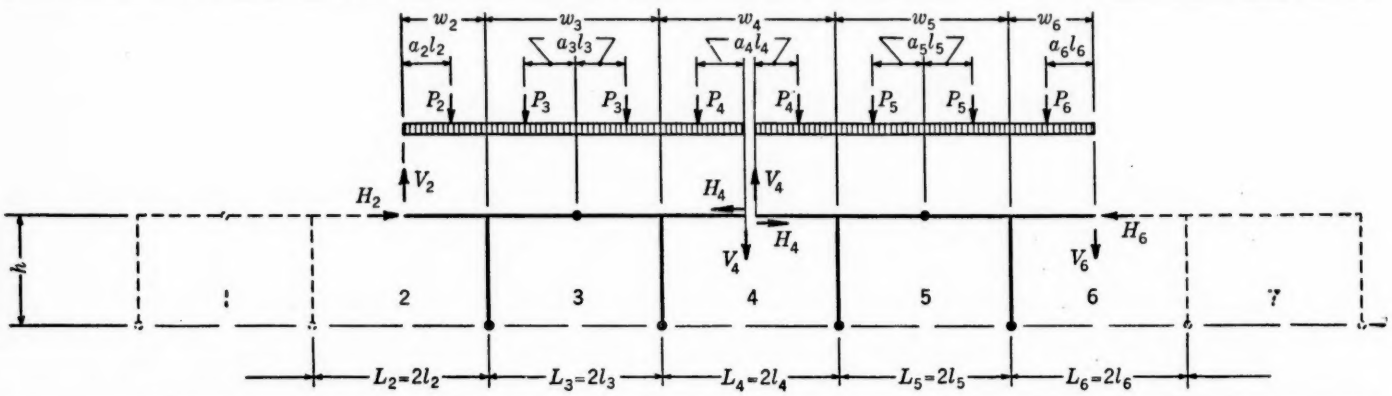
In Eqs. 16 the symbols α and β indicate the total coefficients of the corresponding unknown reactions H or V, obtained by summing up the ($h\,C\,r$)-values or the ($l\,C\,s$)-values in the respective columns of Table 6(a) and Table 7(a), and Q is the total load coefficient, obtained by adding the ($C\,w\,l^2\,t$)-values and the ($C\,l\,P\,t$)-values of the load columns—thus:

$$\left.\begin{aligned}
\alpha_n &= h\,[C_1\, r_{n(n)1} + C_3\, r_{n(n)3} + C_4\, r_{n(n)4} + C_5\, r_{n(n)5} + C_6\, r_{n(n)6} + C_7\, r_{n(n)7}] \\
\beta_n &= l_n\,[C_1\, s_{n(n)1} + C_3\, s_{n(n)3} + C_4\, s_{n(n)4} + C_5\, s_{n(n)5} + C_6\, s_{n(n)6} + C_7\, s_{n(n)7}] \\
\alpha_{n-2} &= h\,[C_1\, r_{n(n-2)1} + C_4\, r_{n(n-2)4} + C_5\, r_{n(n-2)5}] \\
\beta_{n-2} &= l_{n-2}\,[C_1\, s_{n(n-2)1} + C_4\, s_{n(n-2)4} + C_5\, s_{n(n-2)5}] \\
\alpha_{n+2} &= h\,[C_3\, r_{n(n+2)3} + C_6\, r_{n(n+2)6} + C_7\, r_{n(n+2)7}] \\
\beta_{n+2} &= l_{n+2}\,[C_3\, s_{n(n+2)3} + C_6\, s_{n(n+2)6} + C_7\, s_{n(n+2)7}] \\
Q_n &= w_{n+1}\, l^2_{n+1}\,[t_{nw(n+1)6} + t_{nw(n+1)7}) + (1 - a)\, l_{n-1}\, P_{n-1}\, t_{n(n-1)1} + t_{n(n-1)4} + t_{n(n-1)5}]
\end{aligned}\right\} \ldots (17)$$

in which r, s, and t are numerical coefficients, and $C = h\,l\,\frac{k}{I}$ (see Eq. 15a). The symbols α', β', and Q' designate the corresponding values for the V_n-equation, and are similarly obtained from the data in Table 6(b).

The beam framing in each bay (that is, the two beams constituting the horizontal member) is assumed to be symmetrical. In general, there will be little reason for shaping the two wedge beams in a given interior bay differently. However, should the shapes differ, then each beam will have a different C-value. In addition, if the hinge is not located at the center of the span, the r-values and the s-values will also differ for the two beams in the bay. It is also to be noted

TABLE 7.—Contribution of Members to Deflection of a Joint in Interior Bays of Rectangular Frames (Joint 4)



Member	$C^{(a)}$	Coefficients H and V for Δ_{H4} and Δ_{V4}						Load Coefficients for Δ_{H4} and Δ_{V4}									
		$h\,H_2$	$h\,H_4$	$h\,H_6$	$l_2\,V_2$	$l_4\,V_4$	$l_6\,V_6$	$(1-a_2)\,l_2\,P_2$	$(1-a_3)\,l_3\,P_3$	$(1-a_4)\,l_4\,P_4$	$(1-a_5)\,l_5\,P_5$	$(1-a_6)\,l_6\,P_6$	$w_2\,l^2_2$	$w_3\,l^2_3$	$w_4\,l^2_4$	$w_5\,l^2_5$	$w_6\,l^2_6$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)
(a) Coefficients for Horizontal Deflection Δ_H at Joint 4																	
Beam:																	
$3_{L,R}$	C_3	$-\frac{1}{2}$	$\frac{1}{2}$		$-\frac{1}{2}$	$-\frac{1}{2}$		$\frac{1}{2}$		$-\frac{1}{2}$			$\frac{1}{4}$		$-\frac{1}{4}$		
$4_{L,R}$	C_4																
$5_{L,R}$	C_5		$\frac{1}{2}$	$-\frac{1}{2}$		$\frac{1}{2}$	$\frac{1}{2}$			$-\frac{1}{2}$		$\frac{1}{2}$			$-\frac{1}{4}$		$\frac{1}{4}$
Column:																	
2–3	$C_{2,3}$	$-\frac{1}{4}$	$\frac{1}{4}$		$\frac{1}{4}$	$-\frac{1}{4}$		$-\frac{1}{4}$	$\frac{1}{2}$	$-\frac{1}{4}$			$-\frac{1}{8}$	$\frac{1}{4}$	$-\frac{1}{8}$		
3–4	$C_{3,4}$	$-\frac{1}{4}$	$\frac{1}{4}$		$-\frac{1}{4}$	$\frac{1}{4}$		$\frac{1}{4}$	$-\frac{1}{2}$	$\frac{1}{4}$			$\frac{1}{8}$	$-\frac{1}{4}$	$\frac{1}{8}$		
4–5	$C_{4,5}$		$\frac{1}{4}$	$-\frac{1}{4}$		$-\frac{1}{4}$	$\frac{1}{4}$			$\frac{1}{4}$	$-\frac{1}{2}$	$\frac{1}{4}$			$\frac{1}{8}$	$-\frac{1}{4}$	$\frac{1}{8}$
5–6	$C_{5,6}$		$\frac{1}{4}$	$-\frac{1}{4}$		$\frac{1}{4}$	$-\frac{1}{4}$			$-\frac{1}{4}$	$\frac{1}{2}$	$-\frac{1}{4}$			$-\frac{1}{8}$	$\frac{1}{4}$	$-\frac{1}{8}$

TABLE 7.—(*Continued*)

Member	C[a]	Coefficients H and V for Δ_{H4} and Δ_{V4}						Load Coefficients for Δ_{H4} and Δ_{V4}									
		$h\,H_2$	$h\,H_4$	$h\,H_6$	$l_2\,V_2$	$l_4\,V_4$	$l_6\,V_6$	$(1-a_2)\,l_2\,P_2$	$(1-a_3)\,l_3\,P_3$	$(1-a_4)\,l_4\,P_4$	$(1-a_5)\,l_5\,P_5$	$(1-a_6)\,l_6\,P_6$	$w_2\,l^2_2$	$w_3\,l^2_3$	$w_4\,l^2_4$	$w_5\,l^2_5$	$w_6\,l^2_6$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)
(*b*) Coefficient for Vertical Deflection Δ_V at Joint 4																	
Beam:																	
$3_{L,R}$	C_3	$\frac{1}{2}$	$-\frac{1}{2}$		$\frac{1}{2}$	$\frac{1}{2}$		$-\frac{1}{2}$		$\frac{1}{2}$			$-\frac{1}{4}$		$\frac{1}{4}$		
$4_{L,R}$	C_4					2											
$5_{L,R}$	C_5		$\frac{1}{2}$	$-\frac{1}{2}$		$\frac{1}{2}$	$\frac{1}{2}$			$-\frac{1}{2}$		$\frac{1}{2}$			$-\frac{1}{4}$		$\frac{1}{4}$
Column:																	
2–3	$C_{2,3}$	$\frac{1}{4}$	$-\frac{1}{4}$		$-\frac{1}{4}$	$\frac{1}{4}$		$\frac{1}{4}$	$-\frac{1}{2}$	$\frac{1}{4}$			$\frac{1}{8}$	$-\frac{1}{4}$	$\frac{1}{8}$		
3–4	$C_{3,4}$	$-\frac{1}{4}$	$\frac{1}{4}$		$-\frac{1}{4}$	$\frac{1}{4}$		$\frac{1}{4}$	$-\frac{1}{2}$	$\frac{1}{4}$			$\frac{1}{8}$	$-\frac{1}{4}$	$\frac{1}{8}$		
4–5	$C_{4,5}$		$-\frac{1}{4}$	$\frac{1}{4}$		$\frac{1}{4}$	$-\frac{1}{4}$			$-\frac{1}{4}$	$\frac{1}{2}$	$-\frac{1}{4}$			$-\frac{1}{8}$	$\frac{1}{4}$	$-\frac{1}{8}$
5–6	$C_{5,6}$		$\frac{1}{4}$	$-\frac{1}{4}$		$\frac{1}{4}$	$-\frac{1}{4}$			$-\frac{1}{4}$		$-\frac{1}{4}$			$-\frac{1}{8}$	$\frac{1}{4}$	$-\frac{1}{8}$

[a] $C = k\,h\,\frac{l}{I}$; $\quad C' = k\,l_4\,\frac{l}{I}$.

that the components of Q will vary with the loading. For loads other than those shown in the sketch in Table 6, additional components from other affected members will be needed. The simple moment diagrams for some other conditions of loading are shown in Fig. 14. The components of Q due to such loading may be obtained from these diagrams.

Simplified Procedure

For any given framing, r, s, and t can be determined by following the general procedure as outlined and illustrated previously. Once these coefficients are determined, the deflection equations can be set directly. Table 7 contains these coefficients for a typical interior bay of a rectangular framing. The same data for end bays in two arrangements of rectangular framing are given in Tables 8 and 9. With the aid of these tables, it is possible to write the deflection equations of the unknown hinge reactions of a frame without the necessity of supplementary computations. All that is needed for this purpose is a sketch of the frame indicating the properties of the members and the locations of the unknown hinge reaction. Then Eqs. 16 are written by using Table 6 as a guide to formulate the α-coefficient and the β-coefficient defined by Eqs. 17.

Example.—Obtain the deflection equations for the frame shown in Fig. 13(*a*), assuming equal spans and shapes for all members. Omitting C from the expressions (since it has the same value for all members), and using Table 8 for equations of H_2 and V_2,

$$h H_2 (\tfrac{1}{2} + \tfrac{1}{2} + \tfrac{1}{4} + \tfrac{1}{4} + \tfrac{1}{4} + \tfrac{1}{4}) - h H_4 (\tfrac{1}{2} + \tfrac{1}{4} + \tfrac{1}{4}) + 1 V_2 (-\tfrac{1}{2} + \tfrac{1}{2} - \tfrac{1}{4} + \tfrac{1}{4} - \tfrac{1}{4} + \tfrac{1}{4}) + 1 V_4 (\tfrac{1}{2} + \tfrac{1}{4} - \tfrac{1}{4}) + w l^2 (-\tfrac{1}{4} + \tfrac{1}{4}) = 0;$$

$$h H_2 (-\tfrac{1}{2} + \tfrac{1}{2} - \tfrac{1}{4} + \tfrac{1}{4} - \tfrac{1}{4} + \tfrac{1}{4}) - h H_4 (\tfrac{1}{2} - \tfrac{1}{4} + \tfrac{1}{4}) + 1 V_2 (\tfrac{1}{2} + 2 + \tfrac{1}{2} + \tfrac{1}{4} + \tfrac{1}{4} + \tfrac{1}{4} + \tfrac{1}{4}) + 1 V_4 (\tfrac{1}{2} - \tfrac{1}{4} - \tfrac{1}{4}) + w l^2 (\tfrac{1}{4} + \tfrac{1}{4}) = 0.$$

Equations of H_4, V_4, H_6, and V_6 are obtained from Table 7 as follows:

$$-h H_2 (1) + h H_4 (2) - h H_6 (1) - 1 V_2 (\tfrac{1}{2}) + 1 V_4 (0) + 1 V_6 (\tfrac{1}{2}) + w l^2 (0) = 0;$$

$$h H_2 (\tfrac{1}{2}) + h H_4 (0) - h H_6 (\tfrac{1}{2}) - 1 V_2 (0) + 1 V_4 (4) + 1 V_6 (0) - w l^2 (\tfrac{1}{2}) = 0;$$

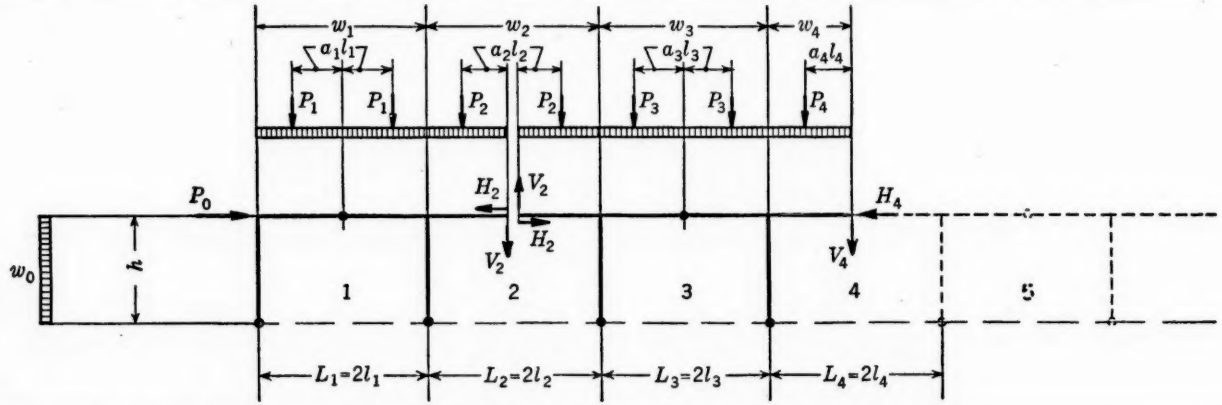
$$-h H_4 (1) + h H_6 (2) - 1 V_4 (\tfrac{1}{2}) + 1 V_6 (0) = 0; \text{ and}$$

$$h H_4 (\tfrac{1}{2}) + h H_6 (0) - 1 V_4 (0) + 1 V_6 (4) = 0.$$

Collecting terms,

$$\left.\begin{array}{l} 2 h H_2 - h H_4 + \tfrac{1}{2} l V_4 = 0 \\ -\tfrac{1}{2} h H_4 + 41 V_2 - \tfrac{1}{2} w l^2 = 0 \\ -h H_2 + 2 h H_4 - h H_6 - \tfrac{1}{2} l V_2 + \tfrac{1}{2} l V_6 = 0 \\ \tfrac{1}{2} h H_2 - \tfrac{1}{2} h H_6 + 41 V_4 - \tfrac{1}{2} w l^2 = 0 \\ -h H_4 + 2 h H_6 - \tfrac{1}{2} l V_4 = 0 \\ \tfrac{1}{2} h H_4 + 41 V_6 = 0 \end{array}\right\} \quad \ldots\ldots\ldots (18)$$

TABLE 8.—Contribution of Members to Deflection of a Joint (Joint 2) in End Bays of Rectangular Frames with an Odd Number of Bays



Member	$C^{(a)}$	Coefficients H and V for Δ_{H2} and Δ_{V2}				Load Coefficients for Δ_{H2} and Δ_{V2}									
		$h\,H_2$	$h\,H_4$	$l_2\,V_2$	$l_4\,V_4$	$h\,P_0$	$(1-a_1)\,l_1\,P_1$	$(1-a_2)\,l_2\,P_2$	$(1-a_3)\,l_3\,P_3$	$(1-a_4)\,l_4\,P_4$	$w_0\,g^2$	$w_1\,l^2_1$	$w_2\,l^2_2$	$w_3\,l^2_3$	$w_4\,l^2_4$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
(a) Coefficients for Horizontal Deflection Δ_{H2} at Joint 2															
Beam:															
$1_{L,R}$..	C_1	$\frac{1}{2}$		$-\frac{1}{2}$		$-\frac{1}{2}$		$-\frac{1}{2}$			$-\frac{1}{4}$		$-\frac{1}{4}$		
$2_{L,R}$..	C_2														
$3_{L,R}$..	C_3	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$			$-\frac{1}{2}$		$\frac{1}{2}$			$-\frac{1}{4}$		$\frac{1}{4}$
Column:															
0–1...	$C_{0,1}$	$\frac{1}{4}$		$-\frac{1}{4}$		$-\frac{1}{4}$	$\frac{1}{2}$	$-\frac{1}{4}$			$[k_w/(2\,k)]-\frac{3}{8}$	$\frac{1}{4}$	$-\frac{1}{8}$		
1–2...	$C_{1,2}$	$\frac{1}{4}$		$\frac{1}{4}$		$-\frac{1}{4}$	$-\frac{1}{2}$	$\frac{1}{4}$			$-\frac{1}{8}$	$-\frac{1}{4}$	$\frac{1}{8}$		
2–3...	$C_{2,3}$	$\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$			$\frac{1}{4}$	$-\frac{1}{2}$	$\frac{1}{4}$			$\frac{1}{8}$	$-\frac{1}{4}$	$\frac{1}{8}$
3–4...	$C_{3,4}$	$\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{4}$			$-\frac{1}{4}$		$-\frac{1}{4}$			$-\frac{1}{8}$		$-\frac{1}{8}$

TABLE 8.—(*Continued*)

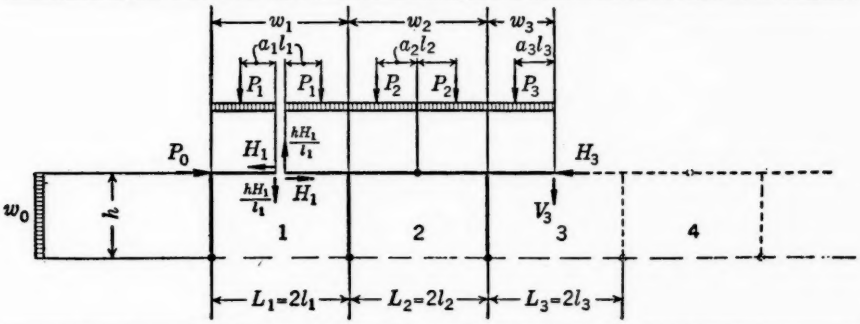
Member	C[a]	Coefficients H and V for Δ_{H2} and Δ_{V2}				Load Coefficients for Δ_{H2} and Δ_{V2}									
		$h\,H_2$	$h\,H_4$	$l_2\,V_2$	$l_4\,V_4$	$h\,P_0$	$(1-a_1)\,l_1\,P_1$	$(1-a_2)\,l_2\,P_2$	$(1-a_3)\,l_3\,P_3$	$(1-a_4)\,l_4\,P_4$	$w_0\,g^2$	$w_1\,l^2_1$	$w_2\,l^2_2$	$w_3\,l^2_3$	$w_4\,l^2_4$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
(*b*) Coefficients for Vertical Deflection Δ_{V2} at Joint 2															
Beam:															
$1_{L,R}$..	C_1	$-\frac{1}{2}$		$\frac{1}{2}$		$\frac{1}{2}$		$\frac{1}{2}$			$\frac{1}{4}$		$\frac{1}{4}$		
$2_{L,R}$..	C_2			2											
$3_{L,R}$..	C_3	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$			$-\frac{1}{2}$		$\frac{1}{2}$			$-\frac{1}{4}$		$\frac{1}{4}$
Column:															
0–1...	$C_{0,1}$	$-\frac{1}{2}$		$\frac{1}{4}$		$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$			$\frac{3}{8}-[k_w/(2\,k)]$	$-\frac{1}{4}$	$\frac{1}{8}$		
1–2...	$C_{1,2}$	$\frac{1}{4}$		$\frac{1}{4}$		$-\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$			$-\frac{1}{8}$	$-\frac{1}{4}$	$\frac{1}{8}$		
2–3...	$C_{2,3}$	$-\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{4}$			$-\frac{1}{4}$	$\frac{1}{2}$	$-\frac{1}{4}$			$-\frac{1}{8}$	$\frac{1}{4}$	$-\frac{1}{8}$
3–4...	$C_{3,4}$	$\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{4}$			$-\frac{1}{4}$	$\frac{1}{2}$	$-\frac{1}{4}$			$-\frac{1}{8}$	$\frac{1}{4}$	$-\frac{1}{8}$

[a] $C = k\,h\,\frac{l}{I}$; $C' = k\,l_2\,\frac{l}{I}$.

Approximate Design

In the selection of sections for an initial or approximate analysis and design, it will be satisfactory to use the simple moments of the component simple frames of an assembly. In most cases, the wedge-beam sections obtained from these moments will not differ materially from those required by the corrected moments of the final or exact analysis. This fact, in itself, constitutes a great

TABLE 9.—Contribution of Members to Deflection of a Joint in End Bays of Rectangular Frames with an Even Number of Bays (Horizontal Deflection at Joint 1)



Member	C[a]	Coefficients[b] H and V			Load Coefficients For Δ_{H1}							
		$h H_1$	$h H_3$	$l_3 V_3$	$h P_0$	$(1-a_1) l_1 P_1$	$(1-a_2) l_2 P_2$	$(1-a_3) l_3 P_3$	$w_0 h^2$	$w_1 l^2_1$	$w_2 l^2_2$	$w_3 l^2_3$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
Beam:												
$1_{L,R}$..	C_1	2			−2				−1			
$2_{L,R}$..	C_2	2	−1	1	−1			1	$-\frac{1}{2}$			$\frac{1}{2}$
$3_{L,R}$..	C_3											
Column:												
0–1...	$C_{0,1}$	1			−1	1			$\frac{k_w}{k}-1$	$\frac{1}{2}$		
1–2...	$C_{1,2}$											
2–3...	$C_{2,3}$	1	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$		1	$-\frac{1}{2}$	$-\frac{1}{4}$		$\frac{1}{2}$	$-\frac{1}{4}$

[a] $C = k\,h\,\frac{l}{I}$. [b] Coefficients H and V for Δ_{H1}.

advantage seldom afforded to the designer in the analysis of rigid or continuous frames. In other words, it is possible to prepare an approximate design of sufficient accuracy with the ease and speed generally obtained in the analysis of statically determinate structures.

Solution of Simultaneous Equations

Perhaps the greatest drawback to the more widespread use of indeterminate structures in general design practice is that the analysis is difficult. The aversion of the average engineer to the so-called "intricacies of refined analysis" and, more particularly, to the task of solving simultaneous equations incidental to the work is well known. Still another adverse factor is the needed accuracy of operations which necessitates the use of computing machines. Since the

analysis of wedge-beam framing remains essentially in the realm of indeterminate structures, the extent of its popularity by most designers will no doubt be subject to the same considerations as those for continuous frames.

The simultaneous equations derived in the analysis of wedge-beam framing are relatively of a simple form. In most cases of design, where the arrangement contains only a few bays, or where the framing is symmetrical and the loading uniform, the group may consist of less than three equations. In such cases the solution of the correspondingly small number of unknowns will involve little difficulty. However, in other arrangements of framing or conditions of loading which result in the setting of a greater number of simultaneous equations, the solution of the unknowns by the use of ordinary methods of computation may become rather burdensome. To obviate this difficulty, and thus render the new framing more acceptable by the average engineer, a more simple procedure is suggested.

Simplified Solution.—Consider the typical expression (see Eq. 16*a*):

$$\alpha_4 H_4 + \beta_{44} V_4 + \alpha_{42} H_2 + \beta_{42} V_2 + \alpha_{46} H_6 + \beta_{46} V_6 + Q_4 = 0 \ldots (19)$$

which was obtained by equating the relations for horizontal deflection at two sides of an interior bay joint. It is characteristic of this expression (as will be noted by examination of data in the corresponding tables) that the coefficient of H_4—that is, the horizontal reaction at the joint under consideration—is always numerically larger than the coefficients of the other unknowns of the equation. Then, for simplicity of reference, the relation can be referred to as the H_4-expression. Furthermore, H_4 can be considered as the principal unknown and the other reactions as the secondary unknowns of the expression. Similarly, each of the latter forces, in turn, will become the principal unknown of the corresponding equation in the group of equations written for a given framing.

In solving for the value of H_4, it will be assumed that its final value, q_4, is obtained by the summation of a series of increments of q, defined as follows:

$$H_4 = q_{41} + q_{42} + q_{43} + \cdots + q_{4n} = q_4 \ldots\ldots\ldots\ldots (20)$$

in which

$$q_{41} = -\frac{Q_4}{\alpha_4} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (21a)$$

$$q_{42} = -\frac{1}{\alpha_4}(\beta_{44} q'_{41} + \alpha_{42} q_{21} + \beta_{42} q'_{21} + \alpha_{46} q_{61} + \beta_{46} q'_{61}) \ldots . (21b)$$

$$q_{43} = -\frac{1}{\alpha_4}(\beta_{44} q'_{42} + \alpha_{42} q_{22} + \beta_{42} q'_{22} + \alpha_{46} q_{62} + \beta_{46} q'_{62}) \ldots . (21c)$$

and

$$q_{4n} = -\frac{1}{\alpha_4}[\beta_{44} q'_{4(n-1)} + \alpha_{42} q_{2(n-1)} + \beta_{42} q'_{2(n-1)} + \alpha_{46} q_{6(n-1)} + \beta_{46} q'_{6(n-1)}] \ldots (21d)$$

In Eqs. 20 and 21, also, $q'_{41}, q'_{42}, \cdots, q'_{4(n-1)}$ are the successive increment values of V_4; $q_{21}, q_{22}, \cdots, q_{2(n-1)}$ are those of H_2, etc., similarly obtained from the respective expression and corresponding cycle of increment operation.

Since the quotient of the numerical coefficient of the secondary unknowns to that of the principal unknown is a small fraction, the values of the successive increments decrease rather rapidly and become negligible quantities in a few

cycles of multiplication. Accordingly, only a few increments will suffice to provide a reasonably accurate solution. Furthermore, the operations will not require the use of computing machines, the desired accuracy being obtained by ordinary slide rule work.

Numerical Example.—Consider the group of six equations (Eqs. 18) derived for the bent in Fig. 13. Here, again it will be assumed that $h = l$, and all members are of like shapes. Then,

$$\left.\begin{aligned} &2H_2 - H_4 + \tfrac{1}{2}V_4 = 0 \\ &4V_2 - \tfrac{1}{2}H_4 + \frac{wl}{2} = 0 \\ &2H_4 - H_2 - H_6 - \tfrac{1}{2}V_2 + \tfrac{1}{2}V_6 = 0 \\ &4V_4 + \tfrac{1}{2}H_2 - \tfrac{1}{2}H_6 - \frac{wl}{2} = 0 \\ &2H_6 - H_4 - \tfrac{1}{2}V_4 = 0 \\ &4V_6 + \tfrac{1}{2}H_4 = 0 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (22)$$

Eqs. 22 can be rewritten in the form:

$$\left.\begin{aligned} H_2 &= \tfrac{1}{2}(H_4 - \tfrac{1}{2}V_4) \\ V_2 &= \tfrac{1}{4}(\tfrac{1}{2}H_4 - \tfrac{1}{2}wl) \\ H_4 &= \tfrac{1}{2}(H_2 + H_6 + \tfrac{1}{2}V_2 - \tfrac{1}{2}V_6) \\ V_4 &= \tfrac{1}{4}(-\tfrac{1}{2}H_2 + \tfrac{1}{2}H_6 + \tfrac{1}{2}wl) \\ H_6 &= \tfrac{1}{2}(H_4 + \tfrac{1}{2}V_4) \\ V_6 &= \tfrac{1}{4}(-\tfrac{1}{2}H_4) \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (23)$$

For initial values of the unknowns, or for the first increments,

$$\left.\begin{aligned} H_{21} &= \tfrac{1}{2}(0) = 0 \\ V_{21} &= \tfrac{1}{4}(-\tfrac{1}{2}wl) = -\tfrac{1}{8}wl \\ H_{41} &= \tfrac{1}{2}(0) = 0 \\ V_{41} &= \tfrac{1}{4}(\tfrac{1}{2}wl) = \tfrac{1}{8}wl \\ H_{61} &= \tfrac{1}{2}(0) = 0 \\ V_{61} &= \tfrac{1}{4}(0) = 0 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (24)$$

From the values in Eqs. 24, the second increments become

$$\left.\begin{aligned} H_{22} &= \tfrac{1}{2}(1 \times 0 - \tfrac{1}{2} \times \tfrac{1}{8}wl) = -\tfrac{1}{32}wl \\ V_{22} &= \tfrac{1}{4}(\tfrac{1}{2} \times 0) = 0 \\ H_{42} &= \tfrac{1}{2}(1 \times 0 + 1 \times 0 + \tfrac{1}{2} \times -\tfrac{1}{8}wl - \tfrac{1}{2} \times 0) = -\tfrac{1}{32}wl \\ V_{42} &= \tfrac{1}{4}(-\tfrac{1}{2} \times 0 + \tfrac{1}{2} \times 0) = 0 \\ H_{62} &= \tfrac{1}{2}(1 \times 0 + \tfrac{1}{2} \times \tfrac{1}{8}wl) = \tfrac{1}{32}wl \\ V_{62} &= \tfrac{1}{4}(-\tfrac{1}{2} \times 0) = 0 \end{aligned}\right\} \quad \ldots (25)$$

Similarly, the third increments, obtained from the second increments, are as follows:

$$\left.\begin{aligned} H_{23} &= \tfrac{1}{2}(1 \times -\tfrac{1}{32}wl - \tfrac{1}{2} \times 0) = -\tfrac{1}{64}wl \\ V_{23} &= \tfrac{1}{4}(\tfrac{1}{2} \times -\tfrac{1}{32}wl) = -\tfrac{1}{256}wl \\ H_{43} &= \tfrac{1}{2}(1 \times -\tfrac{1}{32}wl + 1 \times \tfrac{1}{32}wl + \tfrac{1}{2} \times 0 - \tfrac{1}{2} \times 0) = 0 \\ V_{43} &= \tfrac{1}{4}(-\tfrac{1}{2} \times -\tfrac{1}{32}wl + \tfrac{1}{2} \times \tfrac{1}{32}wl) = \tfrac{1}{1\,28}wl \\ H_{63} &= \tfrac{1}{2}(1 \times -\tfrac{1}{32}wl + \tfrac{1}{2} \times 0) = -\tfrac{1}{64}wl \\ V_{63} &= \tfrac{1}{4}(-\tfrac{1}{2} \times -\tfrac{1}{32}wl) = \tfrac{1}{256}wl \end{aligned}\right\} \quad \ldots (26)$$

Likewise, the fourth, fifth, and sixth increments are obtained:

$$H_{24} = -\frac{1}{512}\,w\,l \qquad H_{25} = -\frac{9}{1{,}024}\,w\,l \qquad H_{26} = -\frac{1}{8{,}192}\,w\,l$$

$$V_{24} = 0 \qquad V_{25} = -\frac{9}{4{,}096}\,w\,l \qquad V_{26} = 0$$

$$H_{44} = -\frac{9}{512}\,w\,l \qquad H_{45} = 0 \qquad H_{46} = -\frac{81}{8{,}192}\,w\,l$$

$$V_{44} = 0 \qquad V_{45} = \frac{1}{2{,}048}\,w\,l \qquad V_{46} = 0$$

$$H_{64} = \frac{1}{512}\,w\,l \qquad H_{65} = -\frac{9}{1{,}024}\,w\,l \qquad H_{66} = \frac{1}{8{,}192}\,w\,l$$

$$V_{64} = 0 \qquad V_{65} = \frac{9}{4{,}096}\,w\,l \qquad V_{66} = 0$$

From the sixth increments it appears that no further operations are needed. The total values, corresponding to six increments, then become

$$\left.\begin{aligned}
H_2 &= \left(0 - \frac{1}{32} - \frac{1}{64} - \frac{1}{512} - \frac{9}{1{,}024} - \frac{1}{8{,}192}\right) w\,l \\
&= -\frac{473}{8{,}192}\,w\,l = -0.05774\,w\,l \\
V_2 &= \left(-\frac{1}{8} + 0 - \frac{1}{256} + 0 - \frac{9}{4{,}096} + 0\right) w\,l \\
&= -\frac{1{,}074}{8{,}192}\,w\,l = -0.13110\,w\,l \\
H_4 &= \left(0 - \frac{1}{32} + 0 - \frac{9}{512} + 0 - \frac{81}{8{,}192}\right) w\,l \\
&= -\frac{481}{8{,}192}\,w\,l = -0.05872\,w\,l \\
V_4 &= \left(\frac{1}{8} + 0 + \frac{1}{128} + 0 + \frac{1}{2{,}048} + 0\right) w\,l = \frac{1{,}092}{8{,}192}\,w\,l = 0.13330\,w\,l \\
H_6 &= \left(0 + \frac{1}{32} - \frac{1}{64} + \frac{1}{512} - \frac{9}{1{,}024} + \frac{1}{8{,}192}\right) w\,l \\
&= \frac{73}{8{,}192}\,w\,l = 0.00891\,w\,l \\
V_6 &= \left(0 + 0 + \frac{1}{256} + 0 + \frac{9}{4{,}096} + 0\right) w\,l \\
&= +\frac{50}{8{,}192}\,w\,l = 0.00610\,w\,l
\end{aligned}\right\} \ldots (27)$$

For further simplification, the necessary operations of the solution may be arranged in a tabular form, as indicated in Fig. 15.

	H_2	V_2	H_4	V_4	H_6	V_6
α_n & β_n	2	4	2	4	2	4
$-\frac{\alpha_{nm}}{\alpha_n}$ or $-\frac{\beta_{nm}}{\alpha_n}$	$\frac{1}{2}H_4$ $-\frac{1}{4}V_4$	$\frac{1}{8}H_4$	$\frac{1}{2}H_2$ $\frac{1}{4}V_2$ $\frac{1}{2}H_6$ $-\frac{1}{4}V_6$	$-\frac{1}{8}H_2$ $\frac{1}{8}H_6$	$\frac{1}{2}H_4$ $\frac{1}{4}V_4$	$-\frac{1}{8}H_4$
$-Q_n$	0	$-\frac{1}{2}wl$	0	$\frac{1}{2}wl$	0	0
$q_{n1}=\frac{-Q_n}{\alpha_n}$	0	$-\frac{1}{8}$	0	$\frac{1}{8}$	0	0
$q_{n2}=-\frac{1}{\alpha_n}\int\alpha_{nm}\cdot q_{n1}$	$\frac{1}{2}\cdot 0=0$ $-\frac{1}{4}\cdot\frac{1}{8}=-\frac{1}{32}$	$\frac{1}{8}\cdot 0=0$	$\frac{1}{2}\cdot 0=0$ $\frac{1}{4}\cdot-\frac{1}{8}=-\frac{1}{32}$ $\frac{1}{2}\cdot 0=0$ $-\frac{1}{4}\cdot 0=0$	$-\frac{1}{8}\cdot 0=0$ $\frac{1}{8}\cdot 0=0$	$\frac{1}{2}\cdot 0=0$ $\frac{1}{4}\cdot\frac{1}{8}=\frac{1}{32}$	$\frac{1}{8}\cdot 0=0$
	$-\frac{1}{32}$	0	$-\frac{1}{32}$	0	$\frac{1}{32}$	0
$q_{n3}=-\frac{1}{\alpha_n}\int\alpha_{nm}\cdot q_{n2}$	$\frac{1}{2}\cdot-\frac{1}{32}=-\frac{1}{64}$ $-\frac{1}{4}\cdot 0=0$	$\frac{1}{8}\cdot-\frac{1}{32}=-\frac{1}{256}$	$\frac{1}{2}\cdot-\frac{1}{32}=-\frac{1}{64}$ $\frac{1}{4}\cdot 0=0$ $\frac{1}{2}\cdot\frac{1}{32}=\frac{1}{64}$ $-\frac{1}{4}\cdot 0=0$	$-\frac{1}{8}\cdot-\frac{1}{32}=\frac{1}{256}$ $\frac{1}{8}\cdot\frac{1}{32}=\frac{1}{256}$	$\frac{1}{2}\cdot-\frac{1}{32}=-\frac{1}{64}$ $\frac{1}{4}\cdot 0=0$	$-\frac{1}{8}\cdot-\frac{1}{32}=\frac{1}{256}$
	$-\frac{1}{64}$	$-\frac{1}{256}$	0	$\frac{1}{128}$	$-\frac{1}{64}$	$\frac{1}{256}$
$q_{n4}=-\frac{1}{\alpha_n}\int\alpha_{nm}\cdot q_{n3}$	0 $-\frac{1}{512}$	0	$-\frac{1}{128}$ $-\frac{1}{1024}$ $-\frac{1}{128}$ $-\frac{1}{1024}$	$\frac{1}{512}$ $-\frac{1}{512}$	0 $\frac{1}{512}$	0
	$-\frac{1}{512}$	0	$-\frac{9}{512}$	0	$\frac{1}{512}$	0
$q_{n5}=-\frac{1}{\alpha_n}\int\alpha_{nm}\cdot q_{n4}$	$-\frac{9}{1024}$ 0	$-\frac{9}{4096}$	$-\frac{1}{1024}$ 0 $\frac{1}{1024}$ 0	$\frac{1}{4096}$ $\frac{1}{4096}$	$-\frac{9}{1024}$ 0	$\frac{9}{4096}$
	$-\frac{9}{1024}$	$-\frac{9}{4096}$	0	$\frac{1}{2048}$	$-\frac{9}{1024}$	$\frac{9}{4096}$
$q_{n6}=-\frac{1}{\alpha_n}\int\alpha_{nm}\cdot q_{n5}$	0 $-\frac{1}{8192}$	0	$-\frac{9}{2048}$ $-\frac{9}{16384}$ $-\frac{9}{2048}$ $-\frac{9}{16384}$	$\frac{9}{8192}$ $-\frac{9}{8192}$	0 $\frac{1}{8192}$	0
	$-\frac{1}{8192}$	0	$-\frac{81}{8192}$	0	$\frac{1}{8192}$	0
$q_n=q_{n1}+q_{n2}+q_{n3}+q_{n4}+q_{n5}+q_{n6}$	$-\frac{473}{8192}wl$ $-.05774wl$	$-\frac{1074}{8192}wl$ $-.13110wl$	$-\frac{481}{8192}wl$ $-.05872wl$	$\frac{1092}{8192}wl$ $.13330wl$	$\frac{73}{8192}wl$ $.00891wl$	$\frac{50}{8192}wl$ $.00610wl$

FIG. 15.—SOLUTION OF SIMULTANEOUS EQUATIONS (FOR THE BENT IN FIG. 11)

Conclusion

The wedge-beam framing system was developed as an economy measure to conserve critical materials of construction. Its presentation here is predicated on that same consideration. It is hoped that full advantage will be taken of the potentialities of the system to assure important savings and, at the same time, to alleviate the present conditions of critical shortage of materials in structural framing. Design formulas and numerical values for typical loadings are presented in Tables 1 to 5.

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